Research Paper

FACE RECOGNITION USING EIGENVECTORS FROM PRINCIPAL COMPONENT ANALYSIS

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ABSTRACT
There are many biometrics methods used for now days for the identification of a person. People in computer vision and pattern recognition have been working on automatic recognition of human faces for the last 20 years. Computer can outperform human in many face recognition through the development technique “eigenfaces”. Particularly those in which large database of faces must be searched. We use principal component analysis with “Eigenface” approach due to its simplicity, speed and learning capability. The design of the face recognition system is based upon “eigenfaces”. The original images of the training set are transformed into a set of eigenfaces \( E \). Then, the weights are calculated for each image of the training set and stored in the set \( W \). Upon observing an unknown image \( Y \), the weights are calculated for that particular image and stored in the vector \( W_Y \). Afterwards, \( W_Y \) is compared with the weights of images, of which one knows for certain that they are facing.

KEY WORDS: Principal component analysis, Eigen Vector, Eigen Value

INTRODUCTION:
Until Kirby and Sirovich [4] applied the Karhunen-Loeve Transform to faces, face recognition systems utilized either feature-based technique, template matching or neural networks to perform the recognition. PCA technique which is provided by Kirby and Sirovich not only resulted in a technique that efficiently represents pictures of faces, but also laid the foundation for the development of the “eigenface” technique of Turk and Pentland [1]. Such patterns, which can be observed in all signals, could be - in the domain of facial recognition - the presence of some objects (eyes, nose, mouth) in any face as well as relative distances between these objects. These characteristic features are called eigenfaces in the facial recognition domain .out of original image data these characteristics can be extracted with the help of a mathematical tool called Principal Component Analysis (PCA). The face space is described by a set of eigenfaces. By projecting a face onto the space expanded by eigenfaces is efficiently represented. Principal component analysis is applied to find the aspects of face which are important for identification. Eigenvectors (eigenfaces) are calculated from the initial face image set. New faces are projected onto the space expanded by eigenfaces and represented by weighted sum of the eigenfaces. To identify faces we make uses of these weights.

Eigenvectors and Eigen values:
We make use of Eigenvectors and Eigenvalues for face recognisition with PCA. So we prepare an initial set of face images \([X_1, X_2, ..., X_n]\). The average face of the whole face distribution is:

\[
X = \frac{(X_1 + X_2 + ... + X_n)}{n}
\]

Then the average face is subtracted from each face,

\[
X_i = X_i - X, \; i = 1, 2, ..., n
\]

\([Y_1, Y_2, ..., Y_n]\) eigenvectors are calculated from the new image set \([X_1', X_2', ..., X_n']\).

\([Y_1, Y_2, ..., Y_n]\) eigenvectors are calculated from the new image set \([X_1', X_2', ..., X_n']\). These eigenvectors are orthonormal to each other. These eigenvectors look like sort of face they do not correspond directly to any face features like eyes, nose and mouth. They are a set of important features which describe the variation in the face image set.

![Figure no. 1](image-url)
Each eigenvector has an eigenvalue associated with it. Eigenvectors on face variation with bigger eigenvalues provide more information than those with smaller eigenvalues. After the eigenfaces are extracted from the covariance matrix of a set of faces, each face is projected onto the eigenface space and represented by a linear combination of the eigenfaces, or has a new descriptor corresponding to a point inside the high dimensional space with the eigenfaces as axes. If we use all the eigenfaces to represent the faces, those in the initial image set can be completely reconstructed. But these eigenfaces are used to represent or code any faces which we try to learn or recognize.

\[ \lambda_i = u_i^T C u_i \]
\[ = \frac{1}{M} u_i^T \sum_{i=1}^{M} \Phi_i \Phi_i^T u_i \]
\[ = \frac{1}{M} \sum_{i=1}^{M} u_i^T \Phi_i \Phi_i^T u_i \]
\[ = \frac{1}{M} \sum_{i=1}^{M} (u_i \Phi_i^T)^T (u_i \Phi_i^T) \]
\[ = \frac{1}{M} \sum_{i=1}^{M} (u_i \Phi_i^T)^2 \]
\[ = \frac{1}{M} \sum_{i=1}^{M} (u_i \Gamma_i^T - \text{mean}(u_i \Gamma_i^T))^2 \]
\[ = \frac{1}{M} \sum_{i=1}^{M} \text{var}(u_i \Gamma_i^T) \]  

(6)

Consider an eigenvector \( \mathbf{u} \) of \( \mathbf{C} \) satisfying the equation

\[ \mathbf{C} = \mathbf{A} \mathbf{A}^T \]  

(6)

Turk and Pentland thus suggest [1] that by selecting the eigenvectors with the largest corresponding eigenvalues as the basis vector, the set of dominant vectors that express the greatest variance are being selected. Recall however, that an N-by-N face image treated as a vector of dimension \( N^2 \) is under consideration. Therefore, if we use the approximated equation derived in Eq. 5, the resultant covariance matrix \( \mathbf{C} \) will be of dimensions \( N^2 \times N^2 \). A typical image of size 256 by 256 would consequently yield a vector of dimension 65,536, which makes the task of determining \( N^2 \) eigenvectors and eigenvalues intractable and computationally unfeasible. Recalling that \( \mathbf{A} = [\Phi_1, \Phi_2, ..., \Phi_m] \), the matrix multiplication of \( \mathbf{A}^T \mathbf{A} \) results in an M-by-M matrix. Since M is the number of faces in the database, the eigenvectors analysis is reduced from the order of the number of pixels in the images \( (N^2) \) to the order of the number of images in the training set (M). In practice, the training set is relatively small (M<< \( N^2 \)) [1], making the computations mathematically manageable.

The simplified method calculates only M eigenvectors while previously it was proven that there are mathematically \( N^2 \) possible eigenvectors. Only the eigenvectors with the largest corresponding eigenvalues from the \( N^2 \) set are selected as the principal components. Thus, the eigenvectors calculated by the alternative algorithm will only be valid, if the resulting eigenvectors correspond to the dominant eigenvectors selected from the \( N^2 \) set.

Consider the eigenvectors, \( \mathbf{v}_i \), of \( \mathbf{A}^T \mathbf{A} \) such that

\[ \mathbf{A}^T \mathbf{v}_i = \lambda_i \mathbf{v}_i \]

Pre-multiplying both sides by \( \mathbf{A} \) and using Eq. (5), we obtain

\[ \mathbf{A} \mathbf{A}^T \mathbf{v}_i = \lambda_i \mathbf{v}_i \]

\[ \mathbf{C} \mathbf{v}_i = \lambda_i \mathbf{v}_i \]

Following this analysis, we construct the M by M matrix \( \mathbf{L} \) of \( \mathbf{A}^T \mathbf{A} \), where

\[ L_{mn} = \Phi_m^T \Phi_n \]

and find the M eigenvectors, \( v_i \) of \( L \). These vectors determine linear combinations of the M training set face images to form the eigenfaces \( u_i = \sum_{k=1}^{M} V_{ik} \Phi_k \).

With this analysis, the calculations are greatly reduced, if the no of data points in face space is less than the dimension of space itself, which in our case is true since \( M << N^2 \), it follows logically that there will only be \( M - 1 \), rather than \( N^2 \), meaningful eigenvectors and so calculation becomes quite manageable. Where \( M \) is no of images in the training set and \( N^2 \) is number of pixels in the image. Thus rather than calculating the \( N^2 \) eigenvectors of \( AA^T \), we can instead compute the eigenvectors of \( A^T A \), and multiply the results with \( A \) in order to obtain the eigenvectors of the covariance matrix, \( C = AA^T \).

Recalling that \( A = [\Phi_1, \Phi_2, \ldots, \Phi_M] \), the matrix multiplication of \( A^T A \) results in an M-by-M matrix. Since \( M \) is the number of faces in the database, the eigenvectors analysis is reduced from the order of the number of pixels in the image ( \( N^2 \) ) to the order of the number of images in the training set (M). In practice, the training set is relatively small (\( M << N^2 \)) \[1\], making the computations mathematically manageable. The simplified method calculates only M eigenvectors while previously it was proven that there are mathematically \( N^2 \) possible eigenvectors. As demonstrated in Eq. (5) only the eigenvectors with the largest corresponding eigenvalues from the \( N^2 \) set are selected as the principal components. Thus, the eigenvectors calculated by the alternative algorithm will only be valid, if the resulting eigenvectors correspond to the dominant eigenvectors selected from the \( N^2 \) set.

**CONCLUSION**

An overview of the design and development of a real-time face recognition system has been presented in this thesis. Although some aspects of the system are still under experimental development, the project has resulted in an overall success, being able to perform reliable recognition in a constrained environment. Under static mode, where recognition is performed on single scaled images without rotation, a recognition accuracy of 96% has been achieved. Face location and normalization were performed in real-time and consistent accuracy in face detection is recorded with video input.

**FUTURE WORK:**

In a research area that is under vigorous investments and major developments, opportunities for future work are abundant. Improvements in the design and implementation of the face detection and normalization modules, or any extensions that could aid in the speed and robustness of the face recognition system are continually in demand.

**REFERENCES**