ABSTRACT
In this paper the optimal placement of UPFC in the power system is being analyzed. Load flow analysis results are obtained and the strength of buses are identified by using stability indices. A comparative study is performed where optimal placement of UPFC is performed by the conventional algorithm and by using Genetic Algorithm.

KEYWORDS FACTS; Load flow analysis; Genetic Algorithm; Power system stability; Unified Power Flow Controllers (UPFC).

I. INTRODUCTION
In the past few decades the power demand has been increasing. With the recent development in the new non-conventional forms of energy and its connection to the power system has increased its complexity in operation and structure. Modern electrical power system is widely interconnected. It minimizes total power generation and fuel cost. Because of the increased power demand some of the transmission lines get over loaded. This causes serious stability issues. Stability is the heart of any system. When two or more generators are connected to the infinite power system synchronization has to be maintained. In the case of any deviation a potential difference is developed between the two machines which cause a voltage drop. Hence voltage frequency and phase angle has to be matched.

Power system stability is the property of the power system which ensures that the system remains in operating equilibrium position through all normal and abnormal operating conditions. Large and sudden changes pose problems in transient stability. Thus it is imperative that transient stability is an important constraint for the optimal load flow. It is necessary to improve the stability while providing maximal transmission loading. This is achieved by controlling the reactive power flow.

With the recent development in FACTS devices it has become possible to control power and enhance the usable capacity of present. Among the FACTS devices unified power flow controller is the most versatile one that can enhance the stability of the system [2]. It can either simultaneously or selectively control both active and reactive flow and bus voltage. UPFC is a combination of shunt and series compensation. It can be replaced with SSSC that injects Voltage (Vr) and a STATCOM that injects current (Ir). Thus UPFC injects power between the two buses to which it is connected. The power flow equations get modified while incorporating UPFC [3]. UPFC can be independently control many parameters. It is reported in many papers that UPFC is an effective FACTS devices for transient stability improvement. In the present paper we are concentrating on the behavior of UPFC in the electrical power system under the influence of fault. Also it is inferred that the stability is improved by optimal placement of UPFC. With the identification of buses experiencing a voltage collapse it is possible to locate the optimal location for UPFC. The suitable location of the UPFC has been evaluated using Voltage Stability Index [7]. A novel heuristic method based on Genetic Algorithm [5] [7] is used to enhance the voltage stability while considering the investment cost and power system losses. The stability enhancement is studied while optimally incorporating UPFC.

II. NEWTON RAPHSON-UPFC ALGORITHM
A. Newton Raphson Method
Power flow analysis is commonly used to determine bus voltage magnitude and phase angle at each bus, real and reactive power flow, power system losses, proper transformer tap settings and circuit loading. It is the most widely used method for power flow analysis. It involves simultaneous solving of non linear algebraic equations by means of successive approximation method. It involves Taylor series expansion. NR method is mathematically superior over other methods. It gives better quadratic convergence and is less prone to divergence with ill conditioned problems. It is suitable for large power system and is more efficient and practical.

Equations Involved:
Current entering in the i\(^{th}\) bus is given by:

\[ I_i = \sum_{j=1}^{n} Y_{ij} V_j \quad \text{(1)} \]

\[ I_i = \sum_{j=1}^{n} |Y_{ij}| |V_j| \angle \theta_{ij} + \delta_j \quad \text{(2)} \]

The complex power at the i\(^{th}\) bus is given by:

\[ P_i - jQ_i = V_i^* I_i \quad \text{(3)} \]

\[ P_i - jQ_i = |V_i| |I_i| \angle -\delta_i \sum_{j=1}^{n} |Y_{ij}| |V_j| \angle \theta_{ij} + \delta_j \quad \text{(4)} \]

Separating the real and imaginary parts,

\[ P_i = \sum_{j=1}^{n} |V_i| |V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \quad \text{(5)} \]

\[ Q_i = -\sum_{j=1}^{n} |V_i| |V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \quad \text{(6)} \]

Expanding above equations in Taylor’s series we get a set of equations which can be summarized in a matrix form as follows.
The diagonal and off-diagonal elements of \( L \) are
\[
\begin{bmatrix}
    K & N \\
    M & L
\end{bmatrix}
\]
\[
\begin{bmatrix}
    \Delta P \\
    \Delta Q
\end{bmatrix}
\rightarrow (7)
\]

Or
\[
\begin{align*}
\frac{\partial \Delta P}{\partial \theta_i} &= \sum_{j \neq i} |V_i||V_j||y_{ij}| \sin(\theta_{ij} - \delta_i - \delta_j) \rightarrow (8) \\
\frac{\partial \Delta P}{\partial \delta_i} &= -|V_i||V_j||y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \quad j \neq i \rightarrow (9) \\
\frac{\partial \Delta P}{\partial \delta_j} &= |V_i||V_j||y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \quad j \neq i \rightarrow (10)
\end{align*}
\]

The output voltage of the series converter is added to the AC terminal voltage \( V_0 \) through the series connected coupling transformer. The injected voltage \( V_{cr} \) acts as an AC series voltage source, changing the effective sending-end voltage as seen from node \( m \). The product of the transmission line current \( I_m \), and the series voltage source \( V_{cr} \) determines the active and reactive power exchanged between the series converter and the AC system.

The real power demanded by the series converter is supplied from the AC power system by the shunt converter through the common DC link. The shunt converter is able to generate or absorb controllable reactive power in both operating modes (i.e. rectifier and inverter). The independently controlled shunt reactive compensation can be used to maintain the shunt converter terminal AC voltage magnitude at a specified value. The equivalent circuit for UPFC is shown in Fig. 2.

### A. UPFC Power Equations

**UPFC Schematic Diagram.**

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**Fig 2: UPFC Equivalent Circuit.**

The equivalent circuit consists of two ideal voltage sources representing the fundamental Fourier series component of the switched voltage waveforms at the AC converter terminals. The ideal voltages sources are:

\[
V_{cr} = V_{cr}(\cos \theta_{cr} + j \sin \theta_{cr}) \rightarrow (16)
\]

\[
V_{ch} = V_{ch}(\cos \theta_{ch} + j \sin \theta_{ch}) \rightarrow (17)
\]

Where \( V_{cr} \) and \( V_{ch} \) are the controllable magnitude and angle of the voltage source representing the shunt converter. They are within limits:

\[
(V_{crmin} \leq V_{cr} \leq V_{crmax}) \quad (0 \leq \theta_{cr} \leq 2\pi)
\]

The magnitude \( V_{ch} \) and angle \( \theta_{ch} \) of the voltage source of the series converter is controlled between limits:

\[
(V_{chmin} \leq V_{ch} \leq V_{chmax}) \quad (0 \leq \theta_{ch} \leq 2\pi), \text{ respectively.}
\]

**A. UPFC Power Equations**

**UPFC Equation for load flow has been discussed in [3] and [4]. Based on the equivalent circuit shown in Fig. 2, the active and reactive power equations are:**

At node k:

\[
P_k = V_k^2 G_{kk} + V_k V_m (G_{km} \cos(\theta_k - \theta_m)) + B_{km} (\sin(\theta_k - \theta_m)) + V_k V_{cr} (G_{kr} \cos(\theta_k - \theta_{cr})) + B_{cr} (\sin(\theta_k - \theta_{cr})) \rightarrow (18)
\]

\[
Q_k = -V_k^2 B_{kk} + V_k V_m (G_{km} \sin(\theta_k - \theta_m)) - B_{km} (\cos(\theta_k - \theta_m)) + V_k V_{cr} (G_{kr} \sin(\theta_k - \theta_{cr})) - B_{cr} (\cos(\theta_k - \theta_{cr})) \rightarrow (19)
\]

At node m:

\[
P_m = V_m^2 G_{mm} + V_m V_k (G_{km} \cos(\theta_m - \theta_k)) + B_{km} (\sin(\theta_m - \theta_k)) + V_m V_{cr} (G_{mm} \cos(\theta_m - \theta_{cr})) + B_{mm} (\sin(\theta_m - \theta_{cr})) \rightarrow (20)
\]

\[
Q_m = -V_m^2 B_{mm} + V_m V_k (G_{km} \sin(\theta_m - \theta_k)) - B_{km} (\cos(\theta_m - \theta_k)) + V_m V_{cr} (G_{mm} \sin(\theta_m - \theta_{cr})) - B_{mm} (\cos(\theta_m - \theta_{cr})) \rightarrow (21)
\]
For series converter:
\[ P_{SR} = V_{CR}^2 G_{mm} + V_{CR} V_{k} (G_{km} \cos(\theta_{CR} - \theta_k) + B_{km} \sin(\theta_{CR} - \theta_k)) + B_{mm} \sin(\theta_{CR} - \theta_m) \]  
\[ Q_{SR} = -V_{CR}^2 B_{mm} + V_{CR} V_{k} (G_{km} \sin(\theta_{CR} - \theta_k) - B_{km} \cos(\theta_{CR} - \theta_k)) + B_{mm} \cos(\theta_{CR} - \theta_m) \]  
\[ \rightarrow (22) \]

For shunt converter:
\[ P_{SR} = -V_{CR}^2 G_{CR} + V_{Sh} V_{k} (G_{km} \cos(\theta_{SR} - \theta_k) + B_{km} \sin(\theta_{SR} - \theta_k)) + B_{mm} \sin(\theta_{SR} - \theta_m) \]  
\[ Q_{SR} = V_{CR}^2 B_{mm} + V_{CR} V_{k} (G_{km} \sin(\theta_{SR} - \theta_k) - B_{km} \cos(\theta_{SR} - \theta_k)) \]  
\[ \rightarrow (23) \]

Where,
\[ Y_{kk} = G_{kk} + j B_{kk} = z_{CR}^{-1} + z_{SR}^{-1} \]  
\[ Y_{mm} = G_{mm} + j B_{mm} = z_{CR}^{-1} \]  
\[ \rightarrow (26) \]
\[ \rightarrow (27) \]

Assuming a free loss converter operation the UPFC neither absorbs nor injects active power with respect to the AC system. The DC link voltage\( V_{dc} \) remains constant. The active power associated with the series converter becomes the DC power\( V_{dc} I_2 \). The shunt converter must supply an equivalent amount of DC power to maintain\( V_{dc} \) constant. Hence, the active power supplied to the shunt converter,\( P_{SR} \) must satisfy the active power demanded by the series converter,\( P_{SR} \), i.e.
\[ P_{SR} + P_{CR} = 0 \]  
\[ \rightarrow (28) \]

B. UPFC Jacobian Equation

The linearized power equations of UPFC are combined with the linearized system of equations corresponding to the rest of the network.
\[ \begin{bmatrix} f(x) \end{bmatrix} = J [\Delta X] \rightarrow (29) \]

Where,
\[ \begin{bmatrix} f(x) \end{bmatrix} = [\Delta P_k \Delta P_m \Delta Q_k \Delta Q_m \Delta P_{mk} \Delta Q_{mk} \Delta P_{bb}]^T \]  
\[ \rightarrow (30) \]

\( \Delta P_{bb} \) is the power mismatch equation given in eqn. 28. \([\Delta X]\) is the solution vector and \([J]\) is the Jacobian matrix. T represents transposition. When UPFC controls voltage magnitude at the shunt converter terminal, i.e. node k, active power injected at node m is PQ-type. The solution vector and Jacobian matrix are given as,
\[ \begin{bmatrix} \Delta X \end{bmatrix} = \begin{bmatrix} \Delta \theta_k \Delta \theta_m \Delta V_{kR} \Delta V_{mR} \Delta V_{CR} \Delta V_{CR} \Delta \theta_{SR} \Delta \theta_{SR} \end{bmatrix} \]  
\[ \rightarrow (31) \]

\[ J = \begin{bmatrix} ... \end{bmatrix} \]  
\[ \rightarrow (32) \]

An Algorithm for the power flow studies using NR method while incorporating UPFC is given in Fig 3.
III. OPTIMAL LOCATION FOR UPFC

In order to find the optimal location for the UPFC to be placed the bus which is mostly affected during faults has to be identified. With the increased loading of transmission and distribution lines, voltage instability problem has become a concern and serious issue for power system planners and operators. The main challenge of this problem is to narrow down the locations where voltage instability could be initiated and to understand the origin of the problem. One effective way to narrow down the workspace is to identify weak buses in the systems, which are most likely to face voltage collapse [6].

A. Voltage Stability Index or L-Index

L-index has been established as a fast voltage stability indicator for transmission system. For an n-bus power system, buses can be separated into two groups: bringing all load buses to the head and denote them as \( \alpha_L \) and putting the generator buses to the tail and term them as \( \alpha_G \). i.e. \( \alpha_L = \{1, 2, ..., n_L-1, n_L\} \) and \( \alpha_G = \{n_L+1, n_L+2, ..., n-1, n\} \) where \( n_L \) is the number of load buses. With a multi-node system, \( I_{bus} = Y_{bus} X V_{bus} \rightarrow (32) \)

By rearranging the nodes we can represent equation (16) as follows
\[
\begin{bmatrix}
I_L \\
I_G
\end{bmatrix} =
\begin{bmatrix}
Y_{11} & Y_{12} \\
Y_{21} & Y_{22}
\end{bmatrix}
\begin{bmatrix}
V_L \\
V_G
\end{bmatrix} \rightarrow (33)
\]

The transmission system itself is linear and allows a representation in terms of a hybrid (H) matrix
\[
\begin{bmatrix}
V_L \\
I_G
\end{bmatrix} = H
\begin{bmatrix}
I_L \\
V_G
\end{bmatrix} \rightarrow (34)
\]

The H-matrix is generated from the Y-matrix by a partial inversion (the voltages \( V_L \) of the consumer nodes in the vector of the unknowns are exchanged against their currents \( I_L \)). For any consumer node \( j \) where, \( j \in \alpha_L \) an equation for \( V_i \) can be derived from the matrix.
\[
\begin{bmatrix}
V_L \\
I_G
\end{bmatrix} =
\begin{bmatrix}
Z_{\alpha L} & F_{\alpha G} \\
K_{\alpha G} & Y_{GG}
\end{bmatrix}
\begin{bmatrix}
I_L \\
V_G
\end{bmatrix} \rightarrow (35)
\]

Where,
- \( V_L, I_L \): Vectors of voltages and currents at consumer nodes
- \( V_G, I_G \): Vectors of voltages and currents at generator nodes
- \( Z_{\alpha L}, F_{\alpha G}, K_{\alpha G}, Y_{GG} \): Sub-matrices of the H-matrix
For any load bus \( j \in \alpha_L \) stability index \( L_j \) can be expressed as
\[
L_j = \frac{S_j^+}{|Y_j^+ V_j|} \rightarrow (36)
\]

Where,
\[
S_j^+ = \left( \sum_{k \in \alpha_G} \frac{Z_{jk}^* S_k}{Z_{jj}^* V_k} \right) V_j \rightarrow (37)
\]

Where, \( S_k \) and \( V_k \) are complex power and complex voltage of node \( k \) respectively. The range of value is \( [0, 1] \). Stability requires that \( L_j < 1 \) and must not be violated on a continuous basis. A global system indicator \( L \) describing the stability of the complete system is \( L = \max (L_j) \). When L approaches 1.0; power system will approach voltage collapse. In practice L must be lower than a threshold value. The predetermined threshold value is specified at the planning stage depending on the system configuration and utility policy regarding quality of service.

IV. GENETIC ALGORITHM BASED NETWORK.

Genetic Algorithms are search techniques, emulating species evolution through generations [5] and [7]. GA is a search algorithm based on the mechanics of natural selection and natural genetics. GA is a robust algorithm that can adaptively search the global optimal point of certain class of engineering problems.

The main steps in the GA procedure are given below:

- **Parameter Set Coding:** Data Reading (Bus/Generator Data) and encode the presence of FACTS device by 1 chromosome. 1 represents the presence of FACTS device and 0 represents the absence of FACTS device in the bus.
- **Initialization:** A first set of solution (individuals) is randomly generated and the GA control parameters are set (number of individuals, number of generations, crossover and mutation rate).
- **Objective function evaluation:** Each solution (individual) is evaluated through the fitness function. This function is related to the objective function and to the constraint expression.
- **Population reproduction:** Individual taking part in successive generations is obtained through the reproductive process is usually composed of three genetic operators: selection, crossover and mutation.
- **Fitness function:** The fitness function is to minimize the generation cost, \( f \) with optimal location. Since GA is applied to maximization problem, minimization of the problem takes the normalized relative fitness value of the population.

A. Selection operator:

It gives preference to better individuals, allowing them to pass on their genes to the next generation. The goodness of each individual depends on its fitness. Fitness may be determined by an objective function or by a subjective judgment.

B. Crossover operator:

Two individuals are chosen from the population using the selection operator. A crossover site along the bit strings is randomly chosen. The values of the two strings are exchanged up to this point. If \( S_1=000000 \) and \( S_2=111111 \) and the crossover point is 2 then \( S_1'=100000 \) and \( S_2'=011111 \). The two new offspring created from this mating are put into the next generation of the population. By recombining portions of good individuals, this process is likely to create even better individuals

C. Mutation operator:

With some low probability, a portion of the new individuals will have some of their bits flipped. Its purpose is to maintain diversity within the population and inhibit premature convergence. Mutation alone
induces a random walk through the search space; Mutation and selection (without crossover) create a parallel, noise-tolerant, hill climbing algorithm.

V. UPFC COST FUNCTION AND FITNESS FUNCTION

The cost function for UPFC is obtained from [7] and [8]:

\[ C_{UPFC} = 0.0003S^2 - 0.2691S + 188.22 \text{ US$/kVAR} \]

Where, \( S \) is operating range of UPFC in MVAR

\[ S = |Q_2 - Q_1| \]

\( Q_1 \) - MVAR flow through the branch before placing FACTS device.

\( Q_2 \) - MVAR flow through the branch after placing FACTS device.

The goal of optimization algorithm is to place FACTS devices in order to enhance voltage stability margin of power system considering cost function FACTS devices. So these devices should be place to prevent congestion in transmission lines and transformer and maintain bus voltages close to their reference. Voltage stability index is used in the objective function considering cost function of UPFC and power system losses. Fitness function is expressed as below:

\[ f(x) = A_1 \text{max}(L_i) + A_2 (\text{Total Investment Cost}) + A_3 (\text{Losses}) \]

\[ \rightarrow (42) \]

The coefficients \( A_1, A_2 \) and \( A_3 \) are optimized by trial and error to 2.78, 0.1 and 2.05 respectively.

VI. RESULTS WITH DISCUSSION

By the conventional NR method the power flow has been carried out for the IEEE 5 Bus system [3]. The location of the UPFC is optimized by conventional method and is compared with the proposed method. L-Index value is computed for the system and it is observed that bus Elm is more sensitive. Hence UPFC is incorporated in between Elm and Main. Another node Elmfa is created. By the conventional NR-Algorithm the UPFC is incorporated in between Lake and Main. Hence another node Lakefa is created.

The voltage magnitude and voltage angle for the system with and without UPFC is summarized in the Table 1.

### Table 1

<table>
<thead>
<tr>
<th>Bus No</th>
<th>Bus</th>
<th>VM</th>
<th>VA</th>
<th>Bus No</th>
<th>VM</th>
<th>VA</th>
<th>Bus No</th>
<th>VM</th>
<th>VA</th>
</tr>
</thead>
<tbody>
<tr>
<td>North</td>
<td>North</td>
<td>1.06</td>
<td>0</td>
<td>North</td>
<td>1.06</td>
<td>0</td>
<td>North</td>
<td>1.06</td>
<td>0</td>
</tr>
<tr>
<td>South</td>
<td>South</td>
<td>1.00</td>
<td>-2.06</td>
<td>South</td>
<td>1.00</td>
<td>0</td>
<td>South</td>
<td>1.00</td>
<td>-2.177</td>
</tr>
<tr>
<td>Lake</td>
<td>Lake</td>
<td>0.987</td>
<td>-4.637</td>
<td>Lake</td>
<td>1.00</td>
<td>-6.020</td>
<td>Lake</td>
<td>0.997</td>
<td>-4.367</td>
</tr>
<tr>
<td>Main</td>
<td>Main</td>
<td>0.984</td>
<td>-4.957</td>
<td>Main</td>
<td>0.984</td>
<td>-2.510</td>
<td>Main</td>
<td>0.996</td>
<td>-4.59</td>
</tr>
<tr>
<td>Elm</td>
<td>Elm</td>
<td>0.972</td>
<td>-5.765</td>
<td>Elm</td>
<td>0.997</td>
<td>-2.319</td>
<td>Elm</td>
<td>1.000</td>
<td>-7.346</td>
</tr>
<tr>
<td>Elmfa</td>
<td>Elmfa</td>
<td>0.975</td>
<td>-4.970</td>
<td>Elmfa</td>
<td>1.020</td>
<td>-4.053</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A comparison of the system with the conventional method to the proposed GA method is given in the Table 2.

### Table 2

|                     | Without UPFC                          | With UPFC\n|---------------------|---------------------------------------|
|                     | Conventional NR-UPFC Algorithm        | Proposed Genetic Algorithm            |
| Total Losses        |                                       |                                       |
| without UPFC        | 12.395 MVA                           | 12.395 MVA                           |
| Total Losses        | 12.100 MVA                           | 12.074 MVA                           |
| Installation Cost   | 184.050 US$/kVAR                      | 183.749 US$/kVAR                      |
| Fitness Function    | 44.012                                | 43.209                                |

The fitness value is shown in Fig 5.


8. [Link to the Siemens PDF on cost-effective reliability]