Research Paper

GEOMETRIC MODELING OF ELLIPTICAL GEAR DRIVES

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ABSTRACT

Gears are used to transmit torque, power and angular velocity in a wide variety of applications as they are compact and have the positive engagement. There are many applications, with irregular rotational motion as in crossed link mechanism, drag link mechanism etc. these mechanisms experience vibration at high speeds because of some undesirable features. The elliptical gear is well known for providing excellent characteristics such as accurate transmission, compact size and ease of dynamic balance. Hence elliptical gears have been successfully used in various types of automatic machinery, packing machines, flying shears, pumps, flow meters and a wide variety of instruments.

In the present work, an attempt has been made to develop a mathematical model of a profile shifted elliptical gear, based on the theory of gearing and the gear generation mechanism. This study also investigates the tooth under cutting of a non-standard elliptical gear, based on the proposed mathematical model. Numerical examples demonstrate the effectiveness of the design process.

Further, a kinematic relationship between the rack cutter and generated gear is framed based on the mathematical model so developed. The geometric properties of the ellipse for the elliptical gear have also been discussed. The mathematical model of the driving and driven profile shifted elliptical gears is also developed. Finally an elliptical gear has been drawn supporting the mathematical equations using computer.

KEYWORDS: Geometric Modeling; Mathematical model; Elliptical Gear rack cutter Elliptical Gear;

1.0 INTRODUCTION

Gears are used transmit torque and angular velocity in a wide variety of applications. Gear are compact, positive engagement, power transmit elements that determines the speed, torque and direction of rotation of driven machine elements. There are wide variety types of gears. This project deals with the simplest type of elliptical gear, where irregular rotational motion requirements are involved. Cams and linkages can provide these special motion requirements as well, but elliptical gears often represent a simpler, more compact and more accurate solution.

Irregular rotational motion is characterized by a repetitive increase and decrease in output-shaft rotational speed for each revolution. Proper application of this irregular motion to another mechanism will alter the output acceleration curves of the original mechanism. Irregular rotation because it is not a start-stop motion, results in a smoother-operating machine, which yields longer life and higher output rates. Elliptical gear can be accurately and economically produced to avoid this irregular rotational motion.

The crossed-link mechanism is kinematically equivalent to an elliptical gear, although the similarity ends there. It is noteworthy that all the undesirable features found in the crossed link mechanism are eliminated when an elliptical gear is used. These gears are interchangeable and can operate on the standard gear center distance as used with circular gear. The crossed link mechanism experiences vibration at high speeds because of unbalance connecting link, and it is nearly impossible to compensate for this vibration. The elliptical gear, while being kinematically equivalent to the crossed link mechanism, does not have this connecting link; as a result, higher speeds may be expected. However, counter balancing of the individual elliptical gear will be required. The cost of manufacture for elliptical gears can be considerably less than or the crossed link but nevertheless will be more than of a comparable circular gear.

Elliptical gears have not been widely uses in industry because of difficulties in their design and manufacture inspite of their usage and application in irregular rotational motion as in flying shears, flow meter etc. Not much work has done in this regard. The geometric equations and properties of the ellipse and mathematical model of the rack cutter and elliptical gear have been discussed in the following lines:

1.1 Elliptical Gear Tooth Nomenclature

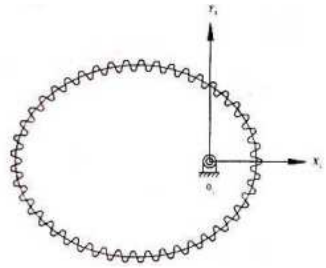

Figure.1: Complete Tooth Profile of an Elliptical Gear

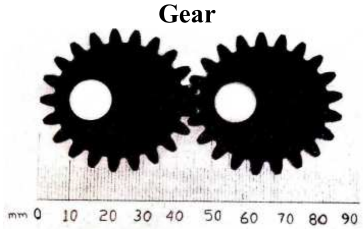

Figure.2: Meshing of Elliptical Gear Pair

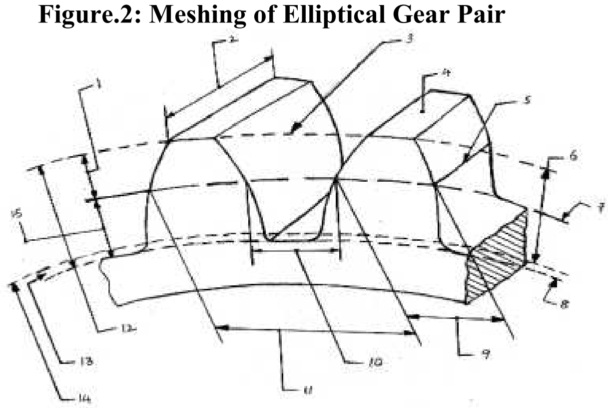

Figure.3: Elliptical Gear Tooth Nomenclature

1) Addendum 2) Face Width 3) Addendum Ellipse 4) Top Land 5) Pitch Surface Element 6) Working Depth 7) Pitch Ellipse 8) Dedendum Ellipse 9) Tooth Thickness 10) Tooth Space 11) Elliptical Pitch 12) Total Depth 13) Clearance 14) Working Depth 15) Dedendum

1.2 Applications of Elliptical Gear

- Hydraulic pumps are used to transform mechanical energy into fluid energy in order to pressurize hydraulic fluid. The most common hydraulic pumps used today are spur gear, internal gear and vane pumps. The clearance of the tooth profile of spur or internal gears and the angular velocity of vane pumps for centrifugal impact limit the pumping of oil. An elliptical gear drive, whose rotation axis coincides with its geometric centre, can also be used as a hydraulic pump. For the same major semi-axis, increasing the eccentricity of an elliptical gear reduces its area. Therefore, the pumping volume of elliptical gear pumps is larger than that of spur or internal gear pumps. Moreover, the angular velocity of elliptical gear pumps can be greater that that of vane pumps.
- Cams and linkages can provide these special motion requirements as well, but noncircular gears often represent a simpler, more compact, or more accurate solution.

  Servo systems may also be able to do the job, but they are usually more expensive and require more expertise to solve motion problems.

  Common requirements handled by noncircular gears include converting a constant input speed into a variable output speed, and providing several different constant-speed segments during an operating cycle. Other applications require combined translation and rotation, or stop-and-dwell motion.
- As with the gears, elliptical gears are used by manufacturers of automatic machines in all applications where variable speed rotary motion is required from a constant speed input.
- Typical applications include machines in the printing and textile industry where a controlled in-feed is required with a rapid return. Elliptical gears are commonly used in packaging and conveyor applications, etc.

1.3 Materials

Elliptical gears can be manufactured in a large range of materials, including aluminum, steel, bronze and stainless steel and our technical department is pleased to collaborate with customers to design and manufacture non-standard gears for special applications.

2.0 MATHEMATICAL MODEL OF THE RACK CUTTER

The generation of elliptical gears can be considered a two-dimensional problem. A standard rack cutter with a complete cutter surface is shown in Fig.3.4a. All methods for manufacturing of elliptical gear can be kinematically considered to consist of a rack cutter that performs pure rolling on the pitch ellipse in the generating process, as shown in Fig 3.5. The shape of the rack cutter consists of two straight lines that from a pressure angle ψ_n with respect to the X_r axis. The circular arcs of radius r with centers at C and D generate the fillet surface of elliptical gears, while the straight line $M_0^{(i)}M_1^{(i)}$ (i=3,4 indicates regions 3and 4 of the rack cutter, respectively) generates tooth surface of the elliptical gears.

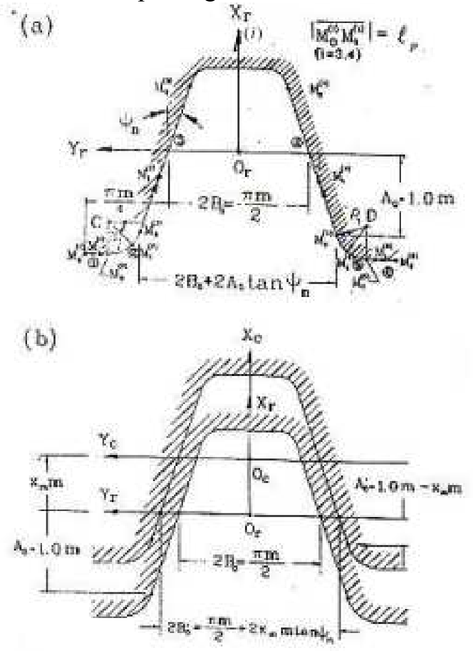

Figure.4: Normal Section of the Rack Cutter Σ_P for Generating the Driving Elliptical Gear (a) Standard Cutter (b) Shifted Cutter.

The mathematical model for the complete tooth profile of elliptical gear is shown below. Coordinate system $S_c(X_c,Y_c), S_1(X_1,Y_1)\ and\ S_f(X_f,Y_f)$ must be set up. The coordinate system S_c, S_1, S_f are attached to the rack cutter, elliptical gear and gear housing, respectively .The contact lines of the gear blank and rack cutter, represented in coordinate system S_c, can be obtained by simultaneously considering the following equations.

$R_1 = [M_{1,c}]R_c^{(i)}$ i=1…..6.

$$\frac{X_c - x_c}{n_{Xc}} = \frac{Y_c - y_c}{n_{Yc}} \quad \text{-------} \quad (1)$$

where i indicates regions 1 to 6 of the corresponding rack cutter surface: X_c and Y_c are coordinates, represented in the coordinate system S_c of the instantaneous center of rotation for the generation mechanism. x_c and y_c are the surface coordinates of the rack cutter. n_{Xc} and n_{Yc} are the direction cosines of the rack cutter surface. The relation show in equation (1) is the equation of meshing. Its relates the surface coordinates $l^{(i)}$ of the rack cutter of motion parameter ϕ_1 of the elliptical gear. The generated elliptical gear tooth surface can be considered a set of contact lines represented in coordinate system,s_1, of rack cutter surface Σ_g and gear blank surface.

2.1 Working Surfaces of the Elliptical Gear

Working surfaces of the elliptical gear are generated by regions 3 and 4 of the rack cutter surface, as shown in Fig.4. According to Equation (1), the working surface of an elliptical gear can be represented by

$$x_1^{(i)} = (-A_0 + l^{(i)}\cos\psi_n)\sin\gamma + (\pm B_0 \pm A_0\tan\psi_n \mp l^{(i)}\sin\psi_c)\cos\gamma + r_1\cos\phi_1 + s\cos\gamma$$
$$y_1^{(i)} = -(-A_0 + l^{(i)}\cos\psi_n)\cos\gamma + (\pm B_0 \pm A_0\tan\psi_n \mp l^{(i)}\sin\psi_c)\sin\gamma - r_1\sin\phi_1 + s\sin\gamma$$
$$l^{(i)} = \frac{A_0}{\cos\psi_n} + B_0\sin\psi_n \pm s\sin\psi_n \qquad \text{----------} \quad (2)$$

Where $l^{(i)} = \left|M_0^{(i)}M_1^{(i)}\right|$. The upper sign indicates the working surface of the elliptical gear generated by region 3 of the rack cutter, and the lower sign represents the working surface of the elliptical gear generated by region 4 of the rack cutter.

3.0 MATHEMATICAL MODEL OF DRIVING AND DRIVEN PROFILE SHIFTED ELLIPTICAL GEAR

3.1 Mathematical Model of Driving Profile-Shifted Elliptical Gears

When two gear surfaces mesh together, both meshing surfaces should remain in tangency during ideal contact conditions. Therefore, conjugate tooth profiles have a common surface normal vector at the contact point which intersects the instantaneous axis of rotation (pitch point I). The equation of meshing for profile-shifted elliptical gears of conjugate shaped teeth can be represented using coordinate system $S_c(X_c,Y_c)$ as follows.

$$\frac{X_c - x_c}{n_{Xc}} = \frac{Y_c - y_c}{n_{Yc}} \qquad \text{--------} \quad (3)$$

$$\frac{l^{(i)}\cos\psi_n - A_0^1}{\sin\psi_n} = \frac{\pm(B_0^1 + A_0^1\tan\psi_n - l^{(i)}\sin\psi_n) + s}{\pm\cos\psi_n} \qquad \text{-----} \quad (4)$$

Solve the equation (4), the equation of meshing for profile-shifted elliptical gear obtained as:

$$\pm S\sin\psi_n = l^{(i)} - \frac{A_0^1}{\cos\psi_n} - B_0^1\sin\psi_n \qquad \text{-------} \quad (5)$$

3.2 Mathematical Model of Driven Profile-Shifted Elliptical Gears

When a profile-shifted elliptical gear drive whose rotation axis coincides with its geometric centre is used in an instrument (e.g. an oil pump), the driving profile-shifted elliptical gear has a major semi-axis a_1, while the driven profile-shifted elliptical gear has a minor semi-axis b_1. Therefore, the distance to the centre of the driving and driven profile-shifted elliptical gears is the total of a_1 and b_1. Hence, if the first generated point of the driving profile-shifted elliptical gear is at the major semi-axis a_1, then the first generated point of the profile-shifted driven elliptical gear should be at the minor semi-axis b_1. Therefore, although the pitch curve of the driven ellipse is the same as that of the driving ellipse, the angle of the driven ellipse leads or lags 90^o compared with that of the driving ellipse. Based on this geometric relation, the pitch curve of the driven ellipse can be represented as follow:

$$r_2(\phi_2) = a_1\sqrt{\frac{1-\varepsilon_1^2}{1-\varepsilon_1^2\sin^2\phi_2}} \qquad \text{----} \quad (6)$$

Where $\phi_2 = \phi_1 + 90^0$. Expressing the pitch curve of the ellipse, $r_2(\phi_2)$, using the Cartesian coordinate system the x_2 and y_2 components along the coordinate axes are

$$x_2 = a_1\sqrt{\frac{1-\varepsilon_1^2}{1-\varepsilon_1^2\sin^2\phi_2}}\cos\phi_2 \qquad \text{-----} \quad (7)$$

and

$$y_2 = -a_1\sqrt{\frac{1-\varepsilon_1^2}{1-\varepsilon_1^2\sin^2\phi_2}}\sin\phi_2 \qquad \text{--------} \quad (8)$$

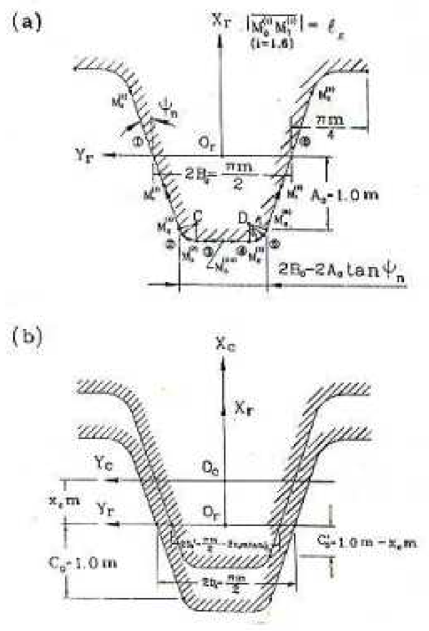

Figure.5: Normal Section of the Rack Cutter Σ_G for Generating the Driven Elliptical Gear (A) Standard Cutter (B) Shifted Cutter

By applying the similar process of sections mathematical model of the rack cutter and kinematic relationship between rack cutter and generated gear are developed, a mathematical model of the driven profile shifted elliptical gear can also be obtained.

$$R_2 = \begin{bmatrix} B_2\sin\gamma_2 + C_2\cos\gamma_2 + r_2\cos\phi_2 + S\cos\gamma_2 \\ -B_2\cos\gamma_2 + C_2\sin\gamma_2 - r_2\sin\phi_2 + S\sin\gamma_2 \end{bmatrix} \qquad \text{----} \quad (9)$$

and

$$\pm S\sin\psi_n = \frac{C_0^1}{\cos\psi_n} - D_0^1\sin\psi_n - l_g \qquad \text{-------} \quad (10)$$

Where

$$B_2 = l_g\cos\psi_n - C_0' \text{ and } C_2 = \pm(D_0' - C_0'\tan\psi_n + l_g\sin\psi_n)$$

and unit normal vector is obtained as follows

$$n_c = \begin{bmatrix} -\sin\psi_n \\ \pm\cos\psi_n \end{bmatrix} \qquad \text{-----} \quad (11)$$

4.0 TOOTH UNDERCUTTING ANALYSIS

Undercut is a result of the generating action of the gear cutting process. The tip of the generating tool sweeps out a trochoidal curve which is required to provide the necessary clearance for the mating member. If this trochoidal curve is modified in any manner, trouble is apt to result. This is especially true when gears operate under metal to metal contact without backlash and are at their nominal size on outside diameter.

The trochoidal curve might be altered in several ways. It can be changed to provide more clearance for the tip of the mating gear by use of a gear shaper cutter that has been sharpened back and has thin teeth. It can be made shallower to produce interference with the tips on the mating gear by the development of a large fillet radius at the top of the generating tool. This often happens as a result of wear, on the leading-in side of the hob or gear shaper cutter, especially in the machining of materials such as stainless steel. Modification of the trochoidal curve can also occur by rolling stock during the shaving operation. In a case of this kind, especially on materials that have poor machinability, the shaving tool has more of a burnishing action than cutting action. As a result, metal is pushed into the clearance area, which was produced by the cutter for the mating gear. The gear "hooks" into the teeth by the interference caused by displaced stock and conjugate action is destroyed. If the teeth were not undercut, the shaving tool would cut away the metal instead of pushing it into an unsupported area.

Disadvantages

- They have a much shorter arc of contact because part of their active profile is removed.
- The undercut is often a source of interference with the tops of the teeth of the mating gear.
- They are considerably weaker than those without undercut.

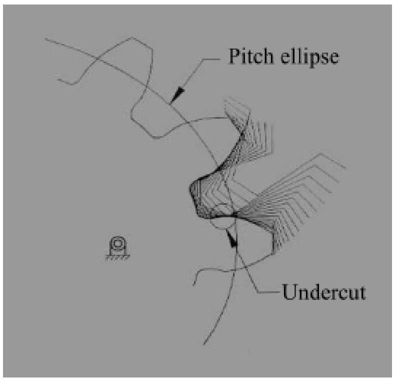

Figure.6: Undercut in an elliptical gear

It is noted that for noncircular gears can obtained the conditions of undercutting as follows:

$$\frac{\partial s}{\partial \gamma_1}\sin^2\psi_n = A_0' - l_p\cos\psi_n \quad \text{-----} \quad (12)$$

5.0 NUMERICAL EXAMPLES AND RESULTS

5.1 Example 1

The shifted rack cutter illustrated in Fig.4b. and Fig.5b. are selected to generate the driving and driven non-standard elliptical gears respectively. Some common design parameters are module m=3.0, pressure angle $\psi_n = 30^0$, A_0 = 1.0m and major semi-axis a_1 =25 mm.

Table 1 presents the relations between the number of teeth, T, the eccentricity, ε_1, the dedendum, the minior axis, the minimum radius, $\rho_{min,}$ and the designed radius, ρ, to prevent tooth under-cutting on the non-standard elliptical gear, when the elliptical gear has a major semi-axis a_1 = 25 mm and m =3.

Table 1 Calculated design parameter corresponding to an elliptical gear with a major semi axis a_1=25mm, ψ_n=30^0, A_0=1.0m and a module m=3.

Number of Teeth(T)	Eccentricity(ε_1)	Minor Axis b(mm)	Dedendum(mm)	ρ_{min} (mm)	ρ (mm)
11	0.962	6.826	3.4	1.86	12.00
12	0.933	8.996	3.4	3.237	12.00
13	0.862	12.672	3.4	6.423	12.00
14	0.752	16.479	3.4	10.862	12.00
15	0.608	19.848	3.4	15.757	12.00
16	0.394	22.977	3.4	21.117	12.00

5.2 Example 2

Another numerical example is considered, in the design parameters of a profile-shifted elliptical gear are the same as in the previous example, except that parameter A_0 = 0.8m.

When parameter A_0 is selected as 0.8m, the tooth profile of elliptical gears is generated with stud teeth. Stud teeth of elliptical gears can be used to reduce pointed teeth tooth undercutting. The condition of tooth undercutting becomes

$$\rho' = \partial S \big/ \partial \gamma_1 = (0.8m - x_m m) \big/ \sin^2 \psi_n \ .$$

According to Table 2, when T=14, the calculated parameters are ε_1=0.752, b_1= 16.479mm, dedendum=3.4mm, ρ_{min}=10.862mm and ρ=9.60mm. Since ρ_{min} exceeds ρ, the driving elliptical gear with stud teeth is produced without tooth undercutting and has no pointed teeth.

Table 2 Calculated design parameter corresponding to an elliptical gear with a major semi axis a_1=25mm, ψ_n=30^0, A_0=0.8m and a module m=3.

Number of Teeth (T)	ρ_{min} (mm)	ρ (mm)
11	1.86	9.60
12	3.237	9.60
13	6.423	9.60
14	10.862	9.60
15	15.757	9.60
16	21.117	9.60

Comparison of Table 1 and 2 reveals that the value of ρ is decreased from 12.000 to 9.600 while parameter A_0 is changed from 1.0 to 0.8m. Additionally, when a smaller A_0 is selected, fewer teeth can be used to generate the tooth profile of elliptical gears without pointed teeth and tooth undercutting

5.3 Example 3

The design parameters of a profile shifted elliptical gear are the same as in Example 2. A negative-shifted modification is used to increase the tooth thickness of the elliptical gear at the addendum circle for T=14.

The tooth thickness of elliptical gear at the addendum circle affects the contact area between the addendum profile of elliptical gear and the pump casing, and a wide tooth thickness can be used. Profile shifted elliptical gears can use a positive-shifted modification to decrease under cutting and increase the tooth thickness at the elliptical pitch. However, an excessively positive shifted modification generates elliptical gears with pointed teeth. Table 3 indicates that tooth under cutting of elliptical gears worsens while negative-shifted modification x_mm increases. When x_mm=0.3 and T=14 the calculated parameters are ε_1=0.752mm, ρ_{min} =10.862mm, ρ=10.80mm. ρ_{min}

exceeds ρ, the non standard elliptical is generated without pointed teeth and tooth undercutting.

Table 3 Calculated design parameter corresponding to a profile shifted elliptical gear with a major semi axis a_1=25mm, $\psi_n=30^0$, A_0=0.8m and a module m=3.

Number of Teeth (T)	$x_m m$	Eccentricity (ε_1)	ρ_{min}(mm)	ρ(mm)
14	0.3	0.752	10.862	10.80
14	0.0	0.752	10.862	12.00
14	-0.3	0.752	10.862	13.20
14	-0.6	0.752	10.862	14.40
14	-0.9	0.752	10.862	15.60
14	-1.2	0.752	10.862	16.80

The numerical examples presented explain how tooth undercutting and pointed teeth can be avoided. This can be clearly observed from the three tables shown above. The relationship between the various terms, viz., number of teeth, eccentricity, minor axis, dedendum are framed on the basis of major axis, pressure angle and module. It is inferred that stud teeth with larger pressure angles can be used to design elliptical gears without tooth undercutting and pointed teeth.

6.0 CONCLUSIONS

A complete mathematical model of elliptical gears, including fillets, bottom lands, and working surfaces of the tooth profile, has been developed in this work. The model shows that the bottom land of an elliptical gear is equidistant from the pitch ellipse. The conditions under which undercutting occurs have been analyzed so as to enable the designers to avoid undercutting elliptical gears in the cutting process, and the constraints on the surface parameters of rack cutters have been investigated.

The mathematical model and undercutting analysis proposed here for elliptical gears should be helpful in the design and production of high-precision elliptical gears. They should also be helpful in the measurement and finite element stress analysis of this type of gearing. A computer program can also been developed to generate elliptical gear tooth surfaces. An elliptical gear drive whose rotation axis coincides with its geometric center can be used in oil pump. The pumping volume of elliptical gear pumps can be enlarged by increasing the eccentricity of the ellipse pitch.

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