AN EOQ MODEL WITH PROGRESSIVE PAYMENT SCHEME UNDER DCF APPROACH WITH PRICE AND CREDIT SENSITIVE DEMAND

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ABSTRACT
In this paper, an EOQ model is developed in which supplier offers the progressive trade credit to the retailer. A progressive trade credit is defined as follows: If the retailer pays the outstanding amount by M, the supplier does not charge any interest. If the retailer pays after M but before N (N>M), the retailer will have to pay interest charges at the rate $Ic$. If the account is settled after N, the retailer will be charged at the rate $Ic$. In this paper the demand of an item depends on the credit period as well as on the price offered by the retailer. Here the retailer’s sales are divided in two categories:

- On cash (which is price sensitive)
- On credit (which is a function of customer’s credit period and price)

The model is developed under Discounted-Cash-Flow (DCF) approach. The present value of all future cash-out-flow is derived for all the three possible scenarios. At the end, a numerical example is given to illustrate the results obtained and sensitivity analysis of various parameters on the optimal solution is carried out.

KEYWORDS: Inventory, Credit-linked demand, Discounted cash flow (DCF) approach, Two-stage credit policy

1. INTRODUCTION
The trade credit period offered by the suppliers to the retailers encourages retailers to buy more and it is a powerful promotional tool that attracts new customers. In the past a lot of work has been done for studying the inventory system behavior under various trade-credit policies offered by the retailer or vendors. Haley and Higgins [8] developed an inventory model to determine EOQ under conditions of permissible delay in payments Goyal [7] presented the similar model with no penalty cost due to late payment Chang [5] then developed an alternative approach to the problem. Chand and ward [4] analysed Goyal’s [7] problem under the assumption of the classical economic order quantity model obtaining different results. Jamal et al [11] & Sarkar et al [13] extended the Goyal’s [7] model by considering the difference between unit price and unit cost. Teng [17] suggested that the buyer should order in similar quantity and take advantages of trade credit frequently Chang et al [5] extended Teng’s [17] work when units in inventory are subject to a constant rate of deterioration. Areclus et al [2] compared scenarios of trade credit and discount for non-deteriorating items. Shah [14] and Agarwal et al [1] extended Goyal’s [7] model to the case of deterioration. All the aforementioned inventory models assumed that the customer would pay for the items as soon as the items are received from the retailer. But in most of the business transactions, the supplier offers a credit period to the retailer and the retailer, in turn passes on some credit period to his customers in order to stimulate his demand. Such a situation where both supplier as well as the retailer offers credit period to their respective customers is known as two-stage credit policy. Recently Huang [9] presented an inventory model assuming that the retailer also offers a credit period to his customer which is shorter than the credit period offered by the supplier. An EOQ model under two stage credit policy when the end demand is price as well as credit period sensitive is developed by Jaggi et al [10]. But progressive payment scheme under due approach is not considered in their paper.

Discounted cash flow (DCF) approach is widely used in business decisions to reflect the time value of money which is neglected in classical EOQ model. It also permits an explicit recognition of the exact timing of each cash flow associated with the inventory system and considers the time value of money as well. Chung [6] presented the discounted cash flow (DCF) approach for the analysis of the optimal inventory policy in the presence of the trade credit. Trippi and Lewin [19], Bensoussan et al [3] gave the DCF/Net present value (NPV) approach for the analysis of the basic EOQ model Kim et al [12] used DCF approach to various inventory systems, Sun & Gueyrame [16] established that the NPV of the total cost in the two approaches, namely average cost and DCF are very close (9.6% difference between their recorder intervals.

In this paper demand depends not only on the price but also on the impact of credit period which is not considered in the above mentioned papers. Here an EOQ model is developed under the DCF approach jointly optimize the retailer’s selling price and replenishment cycle under two-stage credit policy the demand depends on price as well as credit period at the end, a numerical example is given to illustrate the results obtained and sensitivity analysis of various parameters on the optimal solutions is carried out.

2. ASSUMPTIONS
The following assumptions are made to develop the mathematical model

(i) The inventory system deals with single item only.
(ii) Replenishment rate is infinite and instantaneous.
(iii) Lead time is negligible.
(iv) The demand is assumed to be the function of selling price and the length of credit period offered by the retailer. This function has been divided into two parts viz

(a) regular cash-demand which is the function of selling price through out the cycle.

(b) credit period offered by the retailer i.e. during $0 \leq t \leq N$. Hence demand function at any time $t$ can be represented as

$D(t) = \begin{cases} \lambda(p) + R(p,t); & 0 \leq t \leq N_i \\ \lambda(p); & N_i \leq t \leq T \end{cases}$

where we assume that $\lambda(p) = k_p p^e$ where $k_p > 0$ and $e > 1$ while $R(p,t)$ is the demand on
credit during customer’s credit period $N_1$ which is directly proportional to the credit period and inversely proportional to price. We assume that

\[ R(t) = k_1p^{-\gamma}(N_1 - t) \]

where $0 \leq t \leq N_1$, $e > 1$ and $d k_2 > 0$.

(v) The supplier provides a fixed credit period $M$ to settle the account to the retailer and retailer, in turn, passes on a maximum credit period $N_1$ to his customers to settle the account. For simplicity, it is assumed that the customer’s credit period $N_1$ is less than or equal to the retailer’s credit period $M$. It is also assumed that the customer would settle his account only on the last day of the credit period $N_1$ ($N_1 \leq M$).

### Notations:

The following notations are used in this paper:

- $T$: length of replenishment cycle (decision variable)
- $q(t)$: The inventory level at time $t$.
- $Q$: order quantity
- $A$: ordering cost per order
- $c$: unit purchase cost of the item
- $p$: unit selling price of the item
- $h$: holding cost per unit inventory charge
- $I_e$: interest earned per $ per year.
- $M$: retailer’s credit period offered by the supplier for setting the accounts.
- $N_1$: customer’s maximum credit period offered by the retailer, where $N_1 \leq M$.
- $r$: discount rate (opportunity cost) per unit time.
- $I_{c1}$: interest charged per Rs in stock per year by the supplier when the retailer pays after $M$ but before $N$.
- $I_{c2}$: interest charged per Rs in stock per year by the supplier when the retailer pays after $N$ ($I_{c2} > I_{c1}$).
- $Z_1(T, p)$: Retailer’s annual profit for subcase(i) of Case I which is a function of $T$ and $p$.
- $Z_2(T, p)$: Retailer’s annual profit for subcase(ii) of Case I which is a function of $T$ and $p$.
- $Z_3(T, p)$: Retailer’s annual profit for Case II which is a function of $T$ and $p$.
- $PVZ_1(T, p)$: present value of all future cash out flows for subcase(i) of Case I.
- $PVZ_2(T, p)$: present value of all future cash out flows for subcase(ii) of Case I.
- $PVZ_3(T, p)$: present value of all future cash out flows for Case II.

### 3. MATHEMATICAL FORMULATION

Let $q(t)$ be the inventory level at time $t$. A batch of $Q$ units enters the inventory system at the beginning of the cycle. As the time increases, the inventory level decreases rapidly due to cash as well as credit demand i.e. $D(t) = k_1p^{-\gamma} + k_2p^{-\gamma}(N_1 - t)$ up to the period $N_1$. Thereafter, it declines only due to cash demand i.e. $D(t) = k_1p^{-\gamma}$ till the end of the period.

The differential equations governing instantaneous state of $q(t)$ at some instant of time $t$ are given by

\[
\frac{dq_1}{dt} = -\left[k_1p^{-\gamma} + k_2p^{-\gamma}(N_1 - t)\right] \quad (0 \leq t \leq N_1)
\]

satisfying the condition of $q_1(0) = Q$ (1a)

\[
\frac{dq_2}{dt} = -k_1p^{-\gamma} \quad (N_1 \leq t \leq T)
\]

where $q_1(N_1) = q_2(N_1)$ (2a)

Solving (1) we get,

\[
q_1(t) = Q - \frac{k_2p^{-\gamma}N_1^2}{2} - k_1p^{-\gamma}t + \frac{k_2p^{-\gamma}(N_1 - t)^2}{2}
\]

Solving (2) we get,

\[
q_2(t) = k_1p^{-\gamma}(T - t)
\]

The order quantity can be calculated as follows

\[
Q = \int_0^T D(t) \, dt
\]

\[
= k_1p^{-\gamma}T + \frac{k_2p^{-\gamma}N_1^2}{2}
\]

From (3) we get,

\[
q_1(t) = k_1p^{-\gamma}(T - t) + \frac{k_2p^{-\gamma}(N_1 - t)^2}{2}
\]

Therefore, the inventory level at time $t$ during the cycle is

\[
q(t) = \begin{cases} 
q_1(t) = k_1p^{-\gamma}(T - t) + \frac{k_2p^{-\gamma}(N_1 - t)^2}{2}; & (0 \leq t \leq N_1) \\
q_2(t) = k_1p^{-\gamma}(T - t); & (N_1 \leq t \leq T)
\end{cases}
\]
(1) Sales Revenue \( = \frac{pQ}{T} \)  

(2) Cost of placing orders \( = \frac{A}{T} \)  

(3) Cost of purchasing units \( = \frac{cQ}{T} \) 
\[ = \frac{c}{T} \left[ k_1 p^{c-1} T + \frac{k_2 p^{c-1} N_i^2}{2} \right] \]  

(4) Cost of carrying inventory \( = \frac{hT}{I} \left[ \int_{0}^{M} q_1(t)dt + \int_{N_i}^{T} q_2(t)dt \right] \) 
\[ = \frac{Ic}{2T} \left[ k_1 p^{c-1} T^2 + \frac{k_2 p^{c-1} N_i^3}{3} \right] \]  

CASE –I  

Subcase (i) \( (M < N \leq T) \) 

In this case, the retailer deposits the accumulated revenue from cash sales during the period \( (0,M) \) and from credit sales during the period \( (N_i,M) \) into an account that earns interest rate \( I_e \). So in the period \( [0,M] \), the total revenue generated due to cash sales \( \int_{0}^{M} k_1 p^{c-1} dt \) and from credit period sales during the time period \( [N_i,M] \) is 
\[ \int_{N_i}^{M} k_2 p^{c-2} \frac{N_i^2}{2} dt \]. At \( M \), the accounts have to be settled, it is assumed that accounts will be settled by proceeds of sales generated up to \( M \) and by taking a short loan at the interest rate of \( I_{c1} \) and \( I_{c2} \) for the duration of \( (N-M) \) and \( (T-N) \) respectively for financing the unsold stock. 

Consequently, the interest earned per year 
\[ = \frac{I_c}{2T} \left[ k_1 p^{c-1} M^2 + k_2 p^{c-2} \frac{N_i^3}{3} (M - N_i) \right] \]  

The interest payable per year 
\[ = \frac{I_{c1} c}{T} \int_{M}^{T} k_1 p^{c-1} dt + \frac{I_{c2}}{T} \left[ \int_{0}^{M} q_2(t)dt + \int_{N_i}^{T} q_2(t)dt \right] \] 
\[ = I_{c1} c \frac{k_1 p^{c-1} (T-M)^2}{2} + (I_{c2} - I_{c1}) c \frac{k_2 p^{c-2} (T-N)^2}{2T} \]  

Subcase (ii) \( (M < T \leq N) \) 

Here, the interest earned, \( I_{E2} \), during \( [0,M] \) is 
\[ I_{E2} = \frac{I_c}{T} \int_{0}^{M} k_1 p^{c-1} dt + \frac{I_{c2}}{T} \left[ \int_{N_i}^{T} k_2 p^{c-2} \frac{N_i^2}{2} dt \right] \] 
\[ = \frac{I_c}{T} \left[ k_1 p^{c-1} M^2 + k_2 p^{c-2} \frac{N_i^3}{3} (M - N_i) \right] \]  

Buyer has to pay interest per year 
\[ I_{P2} = \frac{c}{T} \int_{M}^{T} q_2(t)dt = \frac{I_{c2} k_1 p^{c-1}}{2T} (T-M)^2 \]  

(15)  

CASE –II \( (M \geq T) \) 

Here, the credit period \( M \) is more or equal to the cycle \( T \), so the retailer earns interest on cash sales during the period \( [0,M] \) and also on credit sales during the period \( [N_i,M] \) but there is no interest payable.  

The interest earned per year 
\[ I_{E3} = \frac{I_c}{T} \int_{0}^{T} k_1 p^{c-1} dt + \frac{I_{c2}}{T} \left[ \int_{N_i}^{T} k_2 p^{c-2} \frac{N_i^2}{2} dt + \int_{T}^{M} Q dt \right] \]
Using the equations (8) to (11), (12) and (13) the retailer’s annual profit for Subcase I $Z_1(T,p)$ can be expressed as

$$
Z_1(T,p) = \frac{1}{2T} \left[ k_1p^{-e}T(2M - T) + k_2p^{-e}N_i^2(M - N_i) \right]
$$

(16)

Using the equations (8) to (11), (12) and (13) the retailer’s annual profit for Subcase I $Z_1(T,p)$ can be expressed as

$$
Z_1(T,p) = \text{Sales revenue-cost of purchasing units-cost of placing orders-cost of carrying inventory+interest earned per year-interest payable per year.}
$$

(17)

The present value of all future cash-out flows is given by

$$
\text{PVZ}_1(T,p) = \sum_{n=0}^{\infty} Z_1(T,p)e^{-nT} = \frac{Z_1(T,p)}{1 - e^{-rT}} = \left(\frac{1}{rT} + \frac{rT}{4}\right)Z_1(T,p)
$$

(18)

where

$$
Z_1(T,p) = \frac{1}{2T} \left[ (p - c)(2k_1p^{-e}T + k_2p^{-e}N_i^2) + I_0p[k_1p^{-e}M^2 + k_2p^{-e}N_i^2(M - N_i)] - 2A \right]
$$

(19)

Using the equations (8) to (11) and (16) the retailer’s annual profit for Subcase (ii) $Z_2(T,p)$ can be expressed as

$$
Z_2(T,p) = \text{Sales revenue-cost of purchasing units-cost of placing orders-cost of carrying inventory+interest earned per year-interest payable per year.}
$$

(20)

The present value of all future cash-out flows is given by

$$
\text{PVZ}_2(T,p) = \sum_{n=0}^{\infty} Z_2(T,p)e^{-nT} = \frac{Z_2(T,p)}{1 - e^{-rT}} = \left(\frac{1}{rT} + \frac{rT}{4}\right)Z_2(T,p)
$$

(21)

where

$$
Z_2(T,p) = \frac{1}{2T} \left[ (p - c)(2k_1p^{-e}T + k_2p^{-e}N_i^2) + I_0p[k_1p^{-e}M^2 + k_2p^{-e}N_i^2(M - N_i)] - 2A \right]
$$

(22)

Using the equations (8) to (11) and (16) the retailer’s annual profit for Case II $Z_3(T,p)$ can be expressed as

$$
Z_3(T,p) = \text{Sales revenue-cost of purchasing units-cost of placing orders-cost of carrying inventory+interest earned per year-interest payable per year.}
$$
The present value of all future cash-out flows is given by

\[
PV_Z(T,p) = \sum_{n=0}^{\infty} Z_3(T,p)e^{-nrT}
\]

\[
= \frac{Z_3(T,p)}{1-e^{-rT}}
\]

\[
= \left(\frac{1}{rT} + \frac{1 + rT}{4}\right)Z_3(T,p)
\]

where

\[
Z_3(T,p) = \frac{1}{2T} \left[ (p-c)(2k_1p^{-z}T + k_2 p^{-z}N^2_i) + I_c p \left[ k_1 p^{-z}T(2M - T) + k_2 p^{-z}N^2_i (M - N_i) \right] - 2A \right]
\]

(23)

Now, for subcase (i) of Case I

\[
\frac{\delta PV_Z_1}{\delta p} = \left(\frac{1}{rT} + \frac{1 + rT}{4}\right) \left[ p^{-z}L_1 + p^{-(e+1)}L_2 \right]
\]

(26)

Where

\[
L_1 = p^{-z} \left( 2k_1 T + k_2 N^2_i (1 - e) + I_c \left( k_1 M^2 + k_2 N^2_i (1 - e)(M - N_i) \right) \right)
\]

(27)

\[
L_2 = he(k_1 T^2 + \frac{k_2 N^2_i}{3}) + I_c e k_2 (T - M)^2 + e(I_c_2 - I_c_1)ck_k(T - M)^2 - ce(2k_1 T + k_2 N^2_i)
\]

(28)

Therefore, \( PV_Z_1(T,p) \) is maximum w.r.t. \( p \).

For Subcase (ii)

\[
\frac{\delta^2 PV_Z_2}{\delta p^2} = \left(\frac{1}{rT} + \frac{1 + rT}{4}\right) p^{-z} \left[ eL_1 + \frac{(e+1)L_2}{p} \right] < 0
\]

(29)

(30)

(31)

(32)
\[ L_2' = \left[ p^{-e+1} \left\{ -2cek_1T + cek_2N_1^2 + hek_1T^2 - \frac{h}{3}k_2N_1^3 + Ic_1cek_2(T-M) \right\} \right] \]  

(33)

\[ \delta^2PVZ_2 = -p^{-e+1} \left\{ eL_1' + (e+1)L_2' \right\} < 0 \]  

(34)

Therefore, \( PVZ_2(T,p) \) is maximum w.r.t. \( p \).

For Case (II) we have

\[ \delta PVZ_3 = \frac{1}{rT} + \frac{rT}{4} \left[ p^{-e}L_1'' + cp^{-e+1}L_2'' \right] \]  

(35)

where

\[ L_1'' = \left\{ 2k_1T(1-e) + k_2N_1^2(1-e) + Ic_1k_1T(2M-T)(1-e) + (1-e)k_2N_1^2(M-N_1) \right\} \]  

(36)

\[ & L_2'' = \left\{ 2ek_1T + ek_1N_2^2 \right\} \]  

(37)

\[ \delta^2PVZ_3 = -p^{-e+1} \left[ eL_1'' + c(e+1)L_2'' \right] < 0 \]  

(38)

Therefore, \( PVZ_3(T,p) \) is maximum w.r.t. \( p \).

**Determination of Optimal time**

Substituting the value of \( p \) in \( Z_i(T,p); i=1,2,3 \) we have the problem of maximizing \( PVZ_i(T); i=1,2,3 \) which now become a function of single variable. To determine optimal time, we have to solve the following mathematical programming problems for two possible cases viz \( M \leq T \) and \( M \geq T \).

**Problem1(P1) (for Subcase (i) of Case I)**

Max \( PVZ_1(T) \)

**Problem2(P2) (for Subcase (ii) of Case I)**

Max \( PVZ_2(T) \)

**Problem3(P3) (for Case II)**

Max \( PVZ_3(T) \)

The optimal values of \( T \) can be calculated using Mathematica.

**Table 1**

<table>
<thead>
<tr>
<th>( M )</th>
<th>( N_1 )</th>
<th>( p^* )</th>
<th>( T^* )</th>
<th>( PVZ(T,p)^* )</th>
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**4. NUMERICAL ANALYSIS**

The parametric values are approximately as \( A=200, k_1=4,00,000, k_2=8,00,000, c=2, Ic_1=0.08, Ic_2=0.10 \)

\( N=70, h=0.08 \)

It is observed from Table 1 that for any fixed \( M \), as \( N_1 \) increases there is decrease in cycle time along with the marginal decrease in unit selling price and also in the total profit. As \( M \) increases for any fixed \( N_1 \) there is marginal decrease in unit price but cycle length, total profit increases implying that it would be economical for the retailer to opt more credit period (\( M \)) and reduce his selling price. From Table-2 it is observed that as the discounting rate \( (r) \) increases, optimum cycle time, unit selling price, \( PVZ(T,p) \) decreases i.e. the developed model is more sensitive to the changes in discounting rate.

**5. CONCLUSION**

In this paper, a mathematical model is developed when the supplier offers progressive trade credit to...
the retailer. The model is developed under DCF approach. Two-stage credit policy with the credit linked demand is considered here. A numerical example is presented to illustrate the theoretical result which suggests that retailer’s should order more and charge fewer price as the retailer’s credit period increases.

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