CRACK DETECTION IN CANTILEVER BEAM BY VIBRATION TECHNIQUES

Kaustubha V. Bhinge, P. G. Karajagi, Swapnil S. Kulkarni

Address for Correspondence
1ME 2nd year Mechanical, 2Asst. Prof. Department of Mechanical Engineering, CAYMET’s Siddhant College of Engineering, Sudumbare, Pune 3Director – Ethika Engineering Solutions (India) Pvt. Ltd., Pune India

ABSTRACT
In this work we try to establish a systematic approach to study and analyze the crack in cantilever beam. This work addresses the inverse problem of assessing the crack location and crack size in various beam structures. The study is based on measurement of natural frequency, a global parameter that can be easily measured at any point conveniently on the structure. The method adopts weightless torsional spring in the beam using the natural frequency due to crack at different location without altering the results. This is the most convenient as it can be measured at any location e.g. service, subsequent frequency measurements could be used to test whether the structure is still sound or not. As the crack propagates in the beam the stiffness of the beam reduces which intern reduces its natural frequency. The analytical equation can be developed using the natural frequency due to crack at different locations & depth along the free length of the beam. The cracks are quiet cumbersome.

INTRODUCTION
Most of rotating machines used in process industries or in manufacturing plant need periodic maintenance and repair. However, failure of just one of these machines can disturb an entire process with loss in terms of production, manpower, and equipment repair or replacement. Also failure of a single machine component in the process industries like petrochemicals or power stations can result into loss of millions of rupees per down time hour. These facts together with higher costs for new equipments have placed increasing demand on plant maintenance to keep existing equipment operating efficiently with higher productivity.

PROBLEM STATEMENT
As discussed above the failure of machine component is loss of time, money and life. Most of the machine components failures are because of the crack. So there is necessity to predict such failures in advance so that losses because of failure are avoided or minimized. Condition based monitoring is one of the preventive maintenance method used in the plant maintenance. So there is requirement to develop the methodology which can be used easily to predict the crack in the machine component from the machine condition such as vibration data.

OBJECTIVE
Objective of this study is to establish a method for detecting the location and depth of a crack in beams using experimental vibration data. The scope of present work is kept limited to case cantilever beam with single crack. The method has been considered only for normal edge crack.

SCOPE OF PROJECT
Here the beam element is taken to carryout vibration analysis, because the beam represents one of the most important structural members in engineering design and construction. The mounting brackets, cranes are the examples of cantilever beam. This method can be used in conditioning based monitoring, which can reduce the loss of time and money.

METHODOLOGY
In this dissertation efforts are made to develop suitable method that can serve as a basis to detect crack location and crack size from measured vibration data. The method based on vibration measurement for detection of location and size of crack is relatively new. Mostly mode frequencies are used for monitoring the crack because modal frequencies are properties of the whole component. The measurements of natural frequencies of machine component at two or more stages of its life offer the possibility of locating damage in the component. If frequencies measured before the component put into service, subsequent frequency measurements could be used to test whether the structure is still sound or not. As the crack propagates in the beam the stiffness of the beam reduces which intern reduces its natural frequency. The analytical equation can be developed using the natural frequency due to crack at different locations & depth along the free length of the beam. The numbers of cases are evaluated & error in the prediction of crack size & location is less than 4%.

LITERATURE SURVEY
S. K. Maiti et al [1][2]
Proposed a method based on measurement of natural frequencies is presented for detection of the location and size of a crack in a stepped cantilever beam. The
crack is represented as a rotational spring and the method involves obtaining plots of its stiffness with crack location for any three natural modes through the characteristic equation. The point of intersection of the three curves gives the crack location. The crack size is then computed using the standard relation between stiffness and crack size. When a crack develops in a component, it leads to a reduction in the stiffness and an increase in its damping. This, in turn, gives rise to a reduction of natural frequencies and a change in the mode shapes. A vibration based method of crack detection utilizes any one of the above as the key parameter. For the vibration based method of crack detection utilizes the characteristic equation. The point of intersection of the three curves gives the crack location. The crack size is then obtained using the relationship between stiffness and crack size a. The accuracy of the method is predicted accurately, the error is again less than 4.5%.

THEORETICAL ANALYSIS

Equation of Motion for the Euler-Bernoulli Beam is as follows,

\[ \frac{\partial^4 y}{\partial x^4} - \lambda^2 \frac{\partial^2 y}{\partial x^2} = 0 \]

Where, \( \lambda^4 = \frac{\rho A \omega^4 L^4}{EI} \) and \( \xi = \beta = x/L \)

\[
|\Delta| = \begin{vmatrix}
1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{vmatrix}
\]

\[
\begin{vmatrix}
\cos \alpha & \cosh \alpha & \sin \alpha & \sinh \alpha \\
-\cos \alpha & \cosh \alpha & -\sin \alpha & \sinh \alpha \\
\frac{K}{\lambda} \sin \alpha - \cosh \alpha & \frac{K}{\lambda} \sinh \alpha + \cosh \alpha & \frac{K}{\lambda} \cos \alpha - \sin \alpha & \frac{K}{\lambda} \cosh \alpha + \cosh \alpha \\
\end{vmatrix}
\]

\[
\begin{vmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
-\cos \lambda & \cosh \lambda & -\sin \lambda & \sinh \lambda \\
\sin \lambda & \sinh \lambda & -\cos \lambda & \cosh \lambda \\
-\cos \alpha & -\cosh \alpha & -\sin \alpha & -\sinh \alpha \\
\cos \alpha & -\cosh \alpha & \sin \alpha & -\sinh \alpha \\
-\sin \alpha & -\sinh \alpha & \cos \alpha & -\cosh \alpha \\
\frac{K}{\lambda} \sin \alpha - \cosh \alpha & \frac{K}{\lambda} \sinh \alpha & \frac{K}{\lambda} \cos \alpha - \sin \alpha & \frac{K}{\lambda} \cosh \alpha + \sinh \alpha \\
\end{vmatrix}
\]

Where, \( \alpha = \lambda \beta \)

The above equation can be written in the form,

\[
\frac{K}{\lambda} |\Delta_1 (\lambda, \beta)| + |\Delta_2 (\lambda, \beta)| = 0
\]

Alternatively it can be written as,

\[
K = -\lambda |\Delta_2| /|\Delta_1|
\]

Figure1 Model of Cracked Cantilever Beam

For the free vibrations of the beam, there is no external excitation and consequently no displacement and no moments at fixed supports.

\[ Y_{1A} = Y_{1B} = 0, \quad \beta = 0 \quad \text{and} \quad Y_{2C} = Y_{2B} = 0, \quad \beta = 1 \]

The continuity conditions at the crack position the displacement, moments and shear forces are

\[ Y_{1B} = Y_{2B}, \quad Y_{1B} = Y_{2B} \]

Substituting the equation \( Y_{1A} \) & \( Y_{1B} \) in above boundary conditions, resulting in a set of eight homogeneous linear algebraic equations for the eight unknown coefficients. From that following characteristic equation is obtained,

From the theory of linear differential equation the solution for above equation is,

\[ Y(x) = a_1 \sin \lambda \beta + a_2 \cos \lambda \beta + a_3 \sinh \lambda \beta + a_4 \cosh \lambda \beta \]

For single crack beam it is considered that the two beam are connected by spring.

\[ Y_1(x) = a_1 \sin \lambda \beta + a_2 \cos \lambda \beta + a_3 \sinh \lambda \beta + a_4 \cosh \lambda \beta \quad 1A \]

\[ Y_2(x) = a_5 \sin \lambda \beta + a_6 \cos \lambda \beta + a_7 \sinh \lambda \beta + a_8 \cosh \lambda \beta \quad 1B \]
Where the explicit expression for $|\Delta|_2$ and $|\Delta|_1$ are,

$$|\Delta|_2 = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\cos \alpha & \cosh \lambda & -\sin \alpha & \sinh \lambda \\ 0 & 0 & 0 & 0 & \sin \lambda & \sinh \lambda & -\cos \alpha & \cosh \lambda \\ \end{vmatrix}$$

$$|\Delta|_1 = \begin{vmatrix} \cos \alpha & \cosh \alpha & \sin \alpha & \sinh \alpha & -\cos \alpha & -\cosh \alpha & -\sin \alpha & -\sinh \alpha \\ -\cos \alpha & \cosh \alpha & -\sin \alpha & \sinh \alpha & \cos \alpha & -\cosh \alpha & \sin \alpha & -\sinh \alpha \\ \sin \alpha & \sinh \alpha & -\cos \alpha & \cosh \alpha & -\sin \alpha & -\sinh \alpha & \cos \alpha & -\cosh \alpha \\ -\sin \alpha & \sinh \alpha & \cos \alpha & \cosh \alpha & \sin \alpha & -\sinh \alpha & -\cos \alpha & -\cosh \alpha \\ \end{vmatrix}$$

For the beam, the first three natural frequencies are measured. Using one of the frequencies & assuming one of the crack locations (e), the non-dimensionalized stiffness $K$ is computed from equation 2. Thereby a variation of stiffness with crack location is obtained. Similar curves can be plotted for other two natural frequencies. Since physically there is only one crack, the position at which the three curves intersect gives crack location & spring stiffness $K$. Further the crack size can be calculated from the following standard relation between stiffness and crack size,

$$K = \frac{b}{72\pi I(a/h)^2} f(a/h)$$

$$f(a/h) = 0.6384 - 1.035(a/h)^2 + 3.7201(a/h)^3 - 5.1773(a/h)^4 + 7.553(a/h)^5 - 7.332(a/h)^6 + 6.799(a/h)^7$$

Where, $b =$ width of beam, $h =$ height of beam & $a =$ crack depth.

CASE STUDY
A cantilever beam with following details is taken for the study.

![Figure 2: Dimension of Cantilever Beam in mm](image)

![Figure 3 FE Model of Uuncracked Cantilever Beam](image)

<table>
<thead>
<tr>
<th>Table 1: Material Details</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Material data</strong></td>
</tr>
<tr>
<td><strong>Young's modulus (E)</strong></td>
</tr>
<tr>
<td><strong>Density (ρ)</strong></td>
</tr>
<tr>
<td><strong>Poisson's Ratio</strong></td>
</tr>
</tbody>
</table>
Finite elements analysis of uncracked and cracked beam is carried out. Normal mode analysis result of uncracked and cracked beam are tabulated in table2. Mode shapes of uncracked beam are shown in figure 4, 5 and 6. As discussed in above using equation 2 a variation of stiffness with crack location is obtained for lowest three transverse natural frequencies. The position at which the three curves intersect gives crack location & spring stiffness K. The plots for different cases are plotted. Refer figure 7 to 11 for the details. Further using equation 3 crack size is calculated. The variation of stiffness with crack location is obtained using MATLAB program.

Figure 4 Uncracked Beam - Mode Shape 1 and 2

Figure 5 Uncracked Beam - Mode Shape 3 and 4

Figure 6 Uncracked Beam - Mode Shape 5 and 6

Table 2: Frequency Results

<table>
<thead>
<tr>
<th>Case No</th>
<th>Natural Frequencies from CAE (rad/s)</th>
<th>ω1</th>
<th>ω2</th>
<th>ω3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncracked beam</td>
<td>584.92 3646.83 10131.45</td>
<td>584.92</td>
<td>3646.83</td>
<td>10131.45</td>
</tr>
<tr>
<td>1</td>
<td>566.54 3645.79 10040.40</td>
<td>566.54</td>
<td>3645.79</td>
<td>10040.40</td>
</tr>
<tr>
<td>2</td>
<td>583.99 3637.42 10114.53</td>
<td>583.99</td>
<td>3637.42</td>
<td>10114.53</td>
</tr>
<tr>
<td>3</td>
<td>570.94 3513.00 9903.00</td>
<td>570.94</td>
<td>3513.00</td>
<td>9903.00</td>
</tr>
<tr>
<td>4</td>
<td>584.02 3598.23 10052.94</td>
<td>584.02</td>
<td>3598.23</td>
<td>10052.94</td>
</tr>
<tr>
<td>5</td>
<td>581.27 3460.08 9845.85</td>
<td>581.27</td>
<td>3460.08</td>
<td>9845.85</td>
</tr>
</tbody>
</table>
Figure 7 Graph of Case-1: Stiffness Vs Crack Location

Figure 8 Graph of Case-2: Stiffness Vs Crack Location

Figure 9 Graph of Case-3: Stiffness Vs Crack Location
CONCLUSION:
Vibration measurement based technique for non-destructively assessing the integrity of structure has certain advantages over the common NDTs, like online conditioning, monitoring etc. It is tedious and time consuming job to apply common NDTs to large structures like long pipelines, rail tracks, aircraft structures, engineering components, etc. An analytical method, which provides the theoretical basis for crack detection using three natural frequencies of uncracked and cracked beam, has been developed for application to uniform beams. The crack is modeled as a torsional spring and is placed at the root of the crack. With this type of modeling the crack location can be predicted accurately. In deriving these theories few important assumptions are made like, the structural member is assumed to behave linearly, the structural properties are assumed to be a time invariant. The error in prediction of crack location and crack size is up to 4%. The proposed method is confirmed by comparing it with results of FEM results. The proposed method is found to be simple and accurate.

### Table 3: Crack Results

<table>
<thead>
<tr>
<th>Case No</th>
<th>Actual</th>
<th>Predicted result</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Location β</td>
<td>Size a/h</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>4</td>
<td>0.6</td>
<td>0.2</td>
</tr>
<tr>
<td>5</td>
<td>0.6</td>
<td>0.4</td>
</tr>
</tbody>
</table>
NOMENCLATURE

a  Crack Depth
b  Width of the Beam
h  Beam Height
a/h  Crack Depth Ratio
L  Beam Length
A  Beam Cross Sectional Area
ρ  Material Density
E  Material Young’s Modulus
υ  Material Poisson’s Ratio
I  Moment of Inertia
ω  Natural Frequency
β  Crack Location
λ  Frequency Parameter
K  Non-Dimensional Stiffness of the Rotational Spring

Representing the Crack

REFERENCES:

8. Nitin Gokhale, Practical finite element analysis
12.