COMPARATIVE STRESS ANALYSIS OF ELLIPTICAL AND CYLINDRICAL PRESSURE VESSEL WITH AND WITHOUT AUTOFRETTAGE CONSIDERATION USING FINITE ELEMENT METHOD

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I. INTRODUCTION
Pressure vessels are leak proof containers. They may be of any shape and range from beverage bottles to the sophisticated ones encountered in engineering construction. The ever-increasing use of vessels for storage, industrial processing and power generation under unusual conditions of pressure, temperature and environment has given special emphasis to analytical and experimental methods for determining their operating stresses.

Pressure vessels are probably the most widespread “machines” within the different industrial sectors. In fact, there is no factory without pressure vessels, steam boilers, tanks, autoclaves, collectors, heat exchangers, pipes, etc. more specifically, pressure vessels represent fundamental components in sectors of enormous industrial importance, such as the nuclear, oil, petro-chemical and chemical sectors. There are periodic international symposia on the problems related to the verification of pressure vessels.

Pressure vessels encountered in nuclear, aerospace and other structures are rotationally symmetric shells subjected to internal pressure. In the design of large rocket motor cases, the number of individual welded segments (viz., head end segment, nozzle end segment, cylindrical segments) will be chosen based on the feasibility of propellant casting, hardware fabrication limits and ease of transportation / handling, etc. These segments may be connected to each other through the tongue and groove type of joints. End domes having central circular openings will be provided at the head end and nozzle end of the motor case. The cylindrical portion of the casing for this type of configuration will be stressed maximum under internal pressure and hence governs the design.

In recent decades, various methods have been proposed for strengthening the vessels. The autofrettage process is possibly the most well-known method. Autofrettage is a process in which the cylinder is subjected to a certain amount of preinternal pressure so that its wall becomes partially plastic. The pressure is then released and the residual stresses lead to a decrease in the maximum von-Mises stress in the working loading stage. This means an increase in the pressure capacity of the cylinder.

Since prediction of structural behavior at the regions of autofrettage in pressure vessels is the ultimate goal of the present study, comparisons are made of different solutions, viz.: finite element solution obtained using ANSYS. The analytical stress results at various junctions of cylinders like outer radius, inner radius and autofrettage radius are found to be in good agreement with FEA results.

II. THEORETICAL BACKGROUND
Cylindrical or spherical pressure vessels (e.g. hydraulic cylinders, gun barrels, pipes, boilers and tanks) are commonly used in industry to carry both liquids and gases under pressure. When the pressure vessels is exposed to this pressure, the material comprising the vessel is subjected to pressure loading, and hence stresses, from all directions. The normal stresses resulting from this pressure are functions of the radius of the element under consideration, the shape of the pressure vessel (i.e., open ended cylinders, closed end cylinders, or sphere) as well as the applied pressure.

A cylindrical pressure with wall thickness t, and inner radius r, is considered (fig 1). A gauge pressure p, exists within the vessel by the working fluid (gas or liquid). For an element sufficiently removed from the ends of the cylinder and oriented as shown in fig 1, two types of normal stress are generated: Tangential stress or hoop stress σ₉, and Longitudinal stress or axial stress σ₉, that both exhibit tension of the material.

Tangential stress or Hoop stress (σ₉): The average stress acting on a cross section area of the vessel. For the hoop stress, consider the pressure vessel section by planes sectioned by plan c in fig 2. A free body diagram of a half segment along with the pressurized working fluid is shown in fig 3. Note that only the loading in the x-direction is shown and that the internal reactions in the material are due to hoop stress acting on incremental areas A, produced by the pressure acting on the project area, Aₚ.
the wall thickness). Fig.5 shows the stress $\sigma_a$, is independent of $r$ (i.e. is constant through the thick wall) and that the axial stress acts uniformly throughout the wall and the pressure acts on the end cap of the cylinder.

The expression for the longitudinal or axial stress is:

$$\sigma_a = \frac{p \times r^2}{(2rt) + t^2}$$

where $r =$ inner radius and $\sigma_a =$ axial stress. Above expressions mentioned are all applicable to only thin walled pressure vessel where thickness is more than 15 times smaller than the vessel’s inner radius. But for Thick walled pressure vessel whose wall thickness is less than 15 times smaller than its inner radius, expressions for Hoop stress and longitudinal stress are different. These are derived using methods developed in a special branch of engineering mechanics called elasticity. Elasticity methods are beyond the scope of the course although the parametric solutions are mathematically exact for the specified boundary conditions are particular problems. For cylindrical pressure vessels subjected to an internal gauge pressure only the following relations result:

$$\sigma_h = \frac{p \times r_0^2}{\pi (r_0^2 - r_i^2)} \left(1 + \frac{r_i^2}{r_0^2} \right)$$

$$\sigma_a = \frac{p \times r_0^2}{(r_0^2 - r_i^2)}$$

$$\sigma_r = \frac{p \times r_0^2}{\pi (r_0^2 - r_i^2)} \left(1 - \frac{r_i^2}{r_0^2} \right)$$

where $r_0 =$ outer radius; $r_i =$ inner radius and $r$ is the radial variable. Equation 3, 4 and 5 are applied for any wall thickness and are not restricted to particular $r/t$ ratio as are the equations 1 and 2. Note that the hoop and radial stress (eh and $\sigma_r$) are functions of $r$, (i.e. vary through the thick wall) and that the axial stress $\sigma_a$, is independent of $r$ (i.e. is constant through the wall thickness). Fig.5 shows the stress distributions through the wall thickness for the hoop and radial stresses. Note that for the radial stress distributions, the maximum and minimum values occur respectively, at the outer wall ($\sigma_r = 0$) and at $\sigma_r = -p$ as noted already for the thin walled pressure vessel.

The expression for the longitudinal or axial stress is:

$$\sigma_a = \frac{p \times r^2}{(2rt) + t^2}$$

Since, this is a thin wall with a small $t$, $\frac{r}{t}$ is smaller and can be neglected such that after simplification,

$$\sigma_a = \frac{p \times r}{2t}$$

The reason why this is possible is that the stress concentration at the inner radius is less than at the bore. Because the outer layers of the tube are also stretched the degree of internal pressure applied during the process is such that they are not stretched beyond their elastic limit. The start point is a single steel tube of internal diameter slightly less than the desired caliber. The tube is subjected to internal pressure of sufficient magnitude to enlarge the bore and in the process the outer layers of the metal are stretched beyond their elastic limit. This means that the inner layers have been stretched to a point where the steel is no longer able to return to its original shape once the internal pressure in the bore has been removed. Although the outer layers of the tube are also stretched the degree of internal pressure applied during the process is such that they are not stretched beyond their elastic limit. Therefore the expansion at the outer layers is less than at the bore. Because the outer layers remain elastic they attempt to return to their original shape; however, they are prevented from doing so completely by the now permanently

**Concept of Autofrettage:** Autofrettage is a metal fabrication technique in which a pressure vessel is subjected to enormous pressure, causing internal portions of the part to yield and resulting in internal compressive residual stresses. The goal of autofrettage is to increase the durability of the final product. Inducing residual compressive stresses into materials can also increase their resistance to stress corrosion cracking; that is, non-mechanically-assisted cracking that occurs when a material is placed in a suitable environment in the presence of residual tensile stress. The technique is commonly used in manufacturing high-pressure pump cylinders, battleship and tank cannon barrels, and fuel injection systems for diesel engines. While some work hardening will occur, that is not the primary mechanism of strengthening.

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stretched inner layers. The effect is that the inner layers of the metal are put under compression by the outer layers in much the same way as though an outer layer of metal had been shrunk on. The next step is to subject the strained inner layers to low temperature heat treatment which results in the elastic limit being raised to at least the autofrettage pressure employed in the first stage of the process. Finally the elasticity of the barrel can be tested by applying internal pressure once more, but this time care is taken to ensure that the inner layers are not stretched beyond their new elastic limit.

When autofrettage is used for strengthening cannon barrels, the barrel is pre-bored to a slightly undersized inside diameter, and then a slightly oversized die is pushed through the barrel. The amount of initial under-bore and size of the die are calculated to strain the material past its elastic limit into plastic deformation, sufficiently far that the final strained diameter is the final desired bore.

**Fig 6: Highly strengthened Pressure vessels**

**III. LITERATURE SURVEY**

Work on Autofrettaging of cylinders started at early year of nineteenth century. Though strong documentation was not available previously but work on this phenomenon was started very early if nineteenth century. In the year of 1960 a remarkable documentation was presented by Farnklin GJ and Morrison JIM [1]. Then in the year of 1986 a very good work on Stress and Deformation Analysis of Autofrettaged High Pressure Vessels was done by Dr. Chen [2]. In the same year Dr. F Kong [3] did a remarkable work on the determination of optimal radius of a thick walled pressure vessel with autofrettage consideration. A scientist named X. L. Gao [4] did an exemplary work on the mathematical modeling of open ended thick cylinder made of Strain-Hardening material in the year of 1992 and the paper was published in a very renowned paper named Journal of Pressure Vessel and Piping. Ruliun Zhu and Jinlai Yang [6] published a paper in the year of 1998 in the International Journal of Pressure Vessel and Piping where they presented an analytic equation of elastic-plastic juncture in autofrettage theory along with a detailed study on load bearing capacity of an autofrettaged cylinder under optimum pressure. In the year of 2003 a very good work was done by a scientist named Nagesh [8] on the analysis of stress distribution of a composite pressure vessel under autofrettage consideration. G.H. Majzoobi and A. Ghomi [9] published a paper at The Journal of Achievements in Materials and Manufacturing Engineering in the year of 2006 on optimization of the weight of compound cylinder for a specific pressure with variables, shrinkage radius and shrinkage tolerance. In their paper they found that the weight of a compound cylinder could reduce by 60% with respect to a single steel cylinder. The reduction was more significant at higher working pressures. While the reduction of weight was negligible for k<2.5, it increased markedly for 2.5<k<5.5. The stress at the internal radii of the outer and inner cylinders became equal to the yield stresses of the materials used for compound cylinders. The experimental results showed higher bursting pressure for optimized cylinders. In the year of 2007 M.H. Hojjati and A. Hassani [10] published a paper at International Journal of Pressure Vessels and Piping, [84 (2007), 310-3] where they studied theoretically and by finite-element modeling the optimum autofrettage pressure and the optimum radius of the elastic–plastic boundary of strain-hardening cylinders in plane strain and plane stress. Equivalent von-Mises stress is used as yield criterion. Comparison of the results of the two methods showed good agreement. Although there was no explicit expression for the optimum autofrettage pressure in plane stress, the equation for plane strain was used with good accuracy. It was also observed that the optimum autofrettage pressure was not a constant value but depended on working pressure. In the year of 2012 Siva Krishna Raparla and T. Seshaiha [11] published a paper on Design and Analysis of Multilayer High Pressure Vessels at International Journal of Engineering Research and Applications (Ijera) (Vol. 2, Issue 1, Pp. 355-361). The main objective of this paper was to design and analysis of multilayer high pressure vessels features of multilayered high pressure vessels, their advantages over mono block vessel were discussed. Various parameters of Solid Pressure Vessel were designed and checked according to the principles specified in American Society of Mechanical Engineers (A.S.M.E) Sec VIII Division 1. The stresses developed in Solid wall pressure vessel and Multilayer pressure vessel was analyzed by using ANSYS, a versatile Finite Element Package. The theoretical values and ANSYS values were compared for both solid wall and multilayer pressure vessels. In the year of 2013 many works were done on the mathematical as well as numerical approach of stress distribution on compound cylinder with or without autofrettage consideration. Ayub A. Miraje and Sunil A. Patil [12] did a remarkable work on Optimum Thickness of Three-Layer Shrink Fitted Compound Cylinder for Uniform Stress Distribution. Bandarupalli Praneeth and T.B.S.Rao [14] did a work on Finite Element Analysis of Pressure Vessel and Piping Design. A very good work was done by Hareram Lohar, Dr. Susenjit Sarkar and Dr. Samar Chandra Mondal [15] on analysis of multi-layered pressure vessel with shrink fit consideration with ANSYS Workbench. A special work was done analytically by Mukhtiar Rana [17] on the autofrettage phenomenon in a thick cylinder in the year of 2013. In his work Mukhtiar Rana solved analytically the stress developed in a thick cylinder during Autofrettaging with help of Matlab. The work has been done as an extension of the work of Mukhtiar Rana and has been implemented on the analysis of an elliptical pressure vessel with autofrettage consideration.

**IV. MATHEMATICAL MODELING OF THICK WALLED PRESSURE VESSEL**

For any thick walled axially symmetric, having plain stress state has the following equations for stress distributions across the thickness derived from lame’s equations:
\[ \frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_h}{r} = 0 \]  
(9)

\[ \varepsilon_r = \frac{\partial u_r}{\partial r} = \left( \frac{1}{E} \right) [\sigma_r - \nu \sigma_h] \]  
(10)

\[ \varepsilon_h = \frac{1}{E} [\sigma_h - \sigma_r] \]  
(11)

Where \( \sigma_r \) is the radial stress; \( \sigma_h \) is the hoop stress; \( E \) is the young’s modulus and \( \nu \) is the poisson’s ratio. \( U_i \) is the deformation (change in directions).

Fig 7: Cylinder under internal pressure

When a cylinder is subjected to an internal pressure \( P_i \), it creates hoop stress and radial stresses. The expression for hoop stress and radial stress are derived from the above mentioned Lame’s equations. If this internal pressure becomes very large, cylinder plate starts yielding from inner radius \( r_i \). There exist a radius \( r_c \) at the elastic-plastic interface where the pressure is \( P_c \). So, the material can be analyzed as region between \( r_i < r < r_c \) and \( r_c < r < r_p \). First one being in plastic state and second one is in elastic state.

Expression for hoop stress, radial stress and axial stress in the elastic region, have already been discussed in ‘Introduction’ chapter. In this chapter expression for hoop stress and radial stress in the region of plastic deformation, would be discussed. The governing equations in formulating stress for elastic-plastic region have been derived by considering power-law hardening model, strain gradient(modified Von Mises) theory for axisymmetric problem.

\[ \sigma_h - \sigma_r = r \frac{\partial \sigma_r}{\partial r} \]  
(12)

\[ r \frac{\partial \varepsilon_h}{\partial r} = \varepsilon_r - \varepsilon_h \]  
(13)

From above equations, employing classical plasticity solution, final useful equations we get is:

\[ \sigma_r = \frac{P_i}{K^2 - 1} \left[ 1 - \frac{r_o^2}{r_i^2} \right] \]  
(14)

\[ \sigma_h = \frac{P_i}{K^2 - 1} \left[ 1 + \frac{r_o^2}{r_i^2} \right] \]  
(15)

Here,

\[ K = \frac{r_o}{r_i} \]

The maximum Von-Misses stress can be determined from the expression below.

\[ \sigma_{von-max} = \frac{\sqrt{3}}{2} (\sigma_h - \sigma_r) \]  
(16)

\[ p_{yo} = \frac{(k^2 - 1)}{\sqrt{3}k^2} \sigma_y \]  
(18)

where

\[ k = \frac{r_o}{r_i} \]

Preliminary pressure \( P_i \) required to start yielding of plate (\( P_i \)) and pressure value required to yield whole plate thickness (\( \sigma_{yield} \)). Expression for \( P_{yo} \) and \( \sigma_{yield} \) are respectively as follows-

\[ P_{yo} = \frac{(k^2 - 1)}{\sqrt{3}k^2} \sigma_y \]  
(17)
stress and Von

\[ m = \frac{r_a}{r_l} \quad \text{and} \quad k = \frac{r_a}{r_l} \]

Except calculating hoop stress and radial stress under the consideration of autofrettage, Von-Misses stress can also be calculated at a given pressure load \( P_w \). Following expression calculate the maximum Von-Misses stress.

\[
\sigma_{\text{von-max}} = \left( \sqrt{3} \varepsilon_n (r_a^2 - r_e^2) + \sqrt{3} \varepsilon_r r_e^2 \left( 1 - \left( \frac{r_e}{r_a} \right)^n \right) + 3 \varepsilon_n r_e \right) \frac{r_e^2}{\varepsilon_0}
\]

(27)

It is worthy to be mentioned over here that, for analysis with autofrettage condition material properties are non-linear and is to be determined by Romberg-Osgood equation as mentioned below.

\[
\sigma = E \varepsilon \left( 1 + \left( \frac{\varepsilon}{\varepsilon_0} \right)^n \right)^{\frac{1}{n}}
\]

(28)

Here,

\[ \varepsilon_0 = \frac{\sigma_{ut}}{E} \]

(29)

\( n \) = material constant.

In the next chapter few Matlab program have been generated out of equations (4.11) through (4.19) to determine different stress values of a thick cylinder considering autofrettaged condition.

VI. VALIDATION OF FEA MODEL

Analysis of a thick walled pressure vessel has been done without considering the case of autofrettaged phenomenon and with consideration of autofrettaged phenomenon. The analysis has been done through a Matlab program using mathematical formulation for calculations of stresses (Hoop stress, Radial stress and Von-Misses stress) in a thick wall pressure vessel without considering autofrettage phenomenon and with consideration of autofrettage phenomenon. These mathematical formulations have already been discussed above. Now outcomes of Matlab program and outcomes of FEA analysis using ANSYS software have been discussed as well as compared.

The above mentioned validation job has been done on a cylinder of following geometrical dimensions material properties.

- Inner radius of the pressure vessel: 300mm
- Wall thickness: 150mm
- Height of the pressure vessel: 600mm
- Young’s modulus of material: 200GPa
- Yield stress: 684MPa
- Poisson’s ratio: 0.3

thick walled cylinder with thickness as a function.

The graphs in figure 8.0 have been validated with a FEA solution using ANSYS software. It is worthy to mention here that, FEA analysis in ANSYS is done in three steps namely, Pre-Processing, Solution and Post-Processing.

In Pre-Processing, modeling of the cylinder has been done. Then it has been discretized into elements and then loading conditions have been imposed.

After generation of meshed model Boundary conditions and Loading have been imposed. Here it is worthy to mention that the boundary conditions have been imposed to the upper and lower edges of the cylinder model and load of 219.39 MPa has been imposed on the inner surface of the cylinder. This value has been calculated with help of equation (17).

After imposition of the boundary conditions and loading conditions the FEA model has been solved in ANSYS through static analysis. When a cylinder is analyzed for Hoop stress, Radial stress and Von-Misses stress without considering autofrettage, static analysis is sufficient and following figures represent the outcomes of the analysis.

![Fig 9: Contour plotting of Hoop Stress distribution in the cylinder.](image)

Above figure depicts the fact that Hoop stress is tensile in nature and maximum at the inner surface and minimum at the outer surface. The variation of the Hoop stress has been represented by a color coding or contour plotting in the model.

Following figure represents the contour plotting of Radial stress distribution and Von-Misses stress distribution in the cylinder respectively.

![Fig 10: Contour plotting of Radial Stress distribution in the cylinder.](image)

![Fig 11: Contour plotting of Von-Misses Stress distribution in the cylinder.](image)

The MATLAB program generates graphs shown in figure 8 shows variation of Hoop stress, Radial stress and Von-misses stress in the above mentioned material constant.

Following expression calculate the maximum Von-Misses stress.

\[
\sigma_{\text{von-max}} = \left( \sqrt{3} \varepsilon_n (r_a^2 - r_e^2) + \sqrt{3} \varepsilon_r r_e^2 \left( 1 - \left( \frac{r_e}{r_a} \right)^n \right) + 3 \varepsilon_n r_e \right) \frac{r_e^2}{\varepsilon_0}
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\]

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thick walled cylinder with thickness as a function.

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![Fig 10: Contour plotting of Radial Stress distribution in the cylinder.](image)

![Fig 11: Contour plotting of Von-Misses Stress distribution in the cylinder.](image)
After representing distribution of Hoop stress, Radial stress and Von-Misses stress through contour plotting, a graphical representation of above three stresses has been shown below as a function of thickness.

**Fig 12: Graphs of Hoop stress, Radial stress and Von-Misses stress.**

It is clearly depicted or perceivable that graphs generated through the Matlab program mentioned above and shown in figure 8 is very much identical with the ANSYS generated graphs shown in figure 12. So it can be said that FEA model for the analysis of thick walled pressure vessel without consideration of autofrettage phenomenon is validated.

Now, a FEA analysis has been done with the same cylinder with autofrettage phenomenon in the following sections. In the FEA analysis of the cylinder with autofrettage consideration, steps up to the imposition of boundary conditions and loading conditions are same as the steps followed in the analysis without autofrettage consideration. After imposition of boundary conditions and loading conditions in the same way as done in the analysis without autofrettage condition, FEA model has been solved by nonlinear analysis to include autofrettage effect. In autofrettage analysis, first the model is analyzed with autofrettage pressure, which calculated by equation (26). Then the same model is analyzed without any loading successively by deleting the autofrettage pressure. This is done because, in autofrettage condition, loading and unloading are done successively. Following figure depicts the reaction forces after the analysis with loading condition.

**Fig 13: Reaction forces after loading conditions.**

**Fig 14: Reaction forces after unloading conditions.**

Above simulations for loading and unloading conditions have been done through nonlinear solution. After the simulation, a working load 200 MPa which is less than the load, calculated from equation (17), needed to start yield is imposed on the inner surface of the cylinder. Following figures show the distribution of Hoop stress, Radial stress and Von-Misses stress by contour plotting. After the contour plotting of Hoop stress, Radial stress and Von-Misses stress, a graph has been plotted as a function of plate thickness.

**Fig 15: Radial stress distribution with autofrettage condition.**

**Fig 16: Hoop stress distribution with autofrettage condition.**

**Fig 17: Von-Misses stress distribution with autofrettage condition.**

**Fig 18: Hoop stress graph with autofrettage condition.**

Now a Matlab program has been generated to validate above graph mathematically by equations (19) to (27). From the Matlab program, following graph is generated. It is very clearly depicted that graph generated by ANSYS and Matlab are same and so it can be concluded that FEA model for simple analysis and analysis with autofrettage condition are mathematically validated.
VII. DESIGN MODIFICATION

In previous section it has been shown that ANSYS is capable of analyzing stresses in circular vessels. In this chapter stresses in an elliptical vessel is to be analyzed with help of ANSYS. Analysis of an elliptical vessel with help of mathematical formulas is very lengthy, critical and cumbersome one. So numerical solution using ANSYS have been approached. Here an elliptical vessel with following dimensions and configurations have been analyzed with help of ANSYS.

- Inner radius of the vessel along x-axis: 300mm
- Eccentricity of the ellipse: \(\frac{3}{4}\)
- This makes inner diameter along y-axis :198mm
- thickness of the plate: 150mm

Foremost a model of the elliptical vessel has been developed in PTC-Creo and it has been converted in IGES format to import in ANSYS. In ANSYS it was then discretized with SOLID186 element. After discretization boundary conditions and pressure onto the inner surface has been imposed. Finally the FEA model of the elliptical cylinder has been solved with ANSYS Sparse solver to get the result.

After simulation, three results have been derived and presented with a contour plotting to show stress distribution among all over volume/masses of the elliptical model.

The three results are,

1. Hoop stress distribution
2. Radial stress distribution

Following figure shows the hoop stress distribution of the elliptical model.

Fig 19: Hoop stress graph generated mathematically by Matlab with autofrettage condition.

Fig 20: Hoop stress distribution the elliptical model.

It is clear from the figure that the maximum hoop stress, tensile as well as compressive are 696 MPa and 387 MPa respectively.

After hoop stress distribution result has been derived for radial stress distribution and a contour plot of that stress distribution has been represented below.

Fig 21: Radial stress distribution the elliptical model.

It is very clear from the above figure that maximum radial stresses, tensile as well as compressive, are 608 MPa and 376 MPa.

Next, Von-Mises stress has been derived and plotted below.

Fig 22: Von Misses stress distribution the elliptical model.

Maximum Von-Misses stresses are 12.6 MPa tensile and 1270 MPa also tensile. Above results are derived not considering the autofrettaged conditions. If autofrettaged conditions are considered then following corresponding results are generated.

The Hoop Stress will be like follows.

Fig 23: Hoop stress distribution the elliptical model considering Autofrettage.

It is seen clearly that Hoop stress distribution within the whole volume of the elliptical vessel part changes than that of the hoop stress distribution derived without considering autofrettage condition. It is clearly depicted that maximum values of Hoop stresses developed in the pressure vessel are quite less if we do autofrettage.

Maximum values of Hoop stress generated or developed in the elliptical vessel due to the consideration of autofrettage are 8.7 MPa in compression and 15.7 MPa in tension. The values were in case of without autofrettage were 387MPa in compression and 696 MPa in tension.

Next Radial stress has been evaluated and represented through a contour plotting as shown below.
It is seen clearly that Radial stress distribution within the whole volume of the elliptical vessel part changes than that of the hoop stress distribution derived without considering autofrettage condition. It is clearly depicted that maximum values of Radial stresses developed in the pressure vessel are quite less if we do autofrettage. Maximum values of Radial stress generated or developed in the elliptical vessel due to the consideration of autofrettage are 8.46 MPa in compression and 13.7 MPa in tension. The values were in case of without autofrettage were 376MPa in compression and 608MPa in tension. Next Von-Misses stress has been evaluated and represented through a contour plotting as shown in figure 25.

It is seen clearly that Von-Misses stress distribution within the whole volume of the elliptical vessel part changes than that of the hoop stress distribution derived without considering autofrettage condition. It is clearly depicted that maximum values of Von-Misses stresses developed in the pressure vessel are quite less if we do autofrettage.

**CONCLUSION AND FUTURE SCOPE**

As per the geometrical configuration it has been calculated that volume of the elliptical pressure vessel is 0.1851m³ and volume of the cylindrical pressure vessel is 0.2036 m³. Hoop stresses developed from the given pressure imposed on circular pressure vessel as well as elliptical pressure vessel has been tabulated below.

<table>
<thead>
<tr>
<th>Sl No.</th>
<th>Stresses</th>
<th>Circular Cross-section</th>
<th>Elliptical Cross-section</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Hoop Stress without Autofrettage</td>
<td>570 MPa (Tensile), 351 MPa (Compressive)</td>
<td>696 MPa (Tensile), 387 MPa (Compressive)</td>
<td>$V_{el} = 1.1 V_{circ}$, $S_{el _tensile} = 0.81 S_{circ _tensile}$, $S_{el _compressive} = 0.90 S_{circ _compressive}$</td>
</tr>
<tr>
<td>2</td>
<td>Hoop Stress with Autofrettage</td>
<td>25.5 MPa (Tensile), 6.61 MPa (Compressive)</td>
<td>15.7 MPa (Tensile), 8.76 MPa (Compressive)</td>
<td>$V_{el} = 1.1 V_{circ}$, $S_{el _tensile} = 1.62 S_{circ _tensile}$, $S_{el _compressive} = 0.75 S_{circ _compressive}$</td>
</tr>
</tbody>
</table>


Note: This Paper/Article is scrutinised and reviewed by Scientific Committee, BITCON-2015, BIT, Durg, CG, India