ABSTRACT

The main focus of this work is to detect fault based on Markov parameter for residual generation and classify fault using Fuzzy classifier. In the first step, from the input-output data, the system is identified using stochastic system identification algorithm. The random system faults and actuator faults are introduced at arbitrary time instants. In the next step, the fault is diagnosed by comparing the Markov parameters of the healthy and faulty systems. Using the residue generated, the fuzzy classifier classifies the fault. The proposed approach is illustrated with a benchmark example and encouraging results are obtained.

KEYWORDS

System identification, Markov parameters, Fault diagnosis, Fuzzy classifier

INTRODUCTION

There has been an increasing interest in fault diagnosis in recent years, as a result of the growing demand for higher performance, efficiency, reliability and safety in control systems. A faulty sensor or actuator may cause process performance degradation, process shut down, or a fatal accident. Quick fault detection and isolation can help avoid abnormal event progression and minimize the quality and productivity offsets [1]. The particular feature of fault diagnosis is using the closed-loop monitoring information in control system to establish the quantitative and qualitative process model, detecting and then isolating the main failures in sensors, actuators, and the controlled process[7],[8].

Since the sensors, actuators and controlled process are often positioned in a harsh field, medium erosion, changes in temperature and humidity, electromagnetic disturbance, lightning strike, human errors and other factors would easily get the components performance degraded or damaged, which are the main sources of failures in control systems [10]. The highly reliable controller in control system establishes the foundation for the fault diagnosis in which the software redundancy could take place of the hardware redundancy [11][13]. There is an abundance of literature on research work for fault diagnosis. Prof. Frank (Duisburg University, German) classified fault diagnosis methods into three general categories, which include the knowledge based methods, analytical model based methods and signal based methods [16]. Fault detection and fault isolation are two important stages in the process of Fault Diagnosis in Control Systems (FDCS)[6][9].
In this paper analytical model based method is used for fault diagnosis since it is a cheaper way to enhance dependability of system than other methods [4], [5]. The Markov parameter is used to detect the fault in a linear dynamical system. The approach basically consists of identifying a system by forming a Hankel matrix from input and output pairs [3]. Using the sliding window approach the system is identified online. Parameter deviations are introduced arbitrarily in between and the faulty system is identified. The residue for the fault classification is obtained by comparing the Markov parameters of faulty and non faulty systems [2]. The fuzzy inference system for known fault conditions serves as a classifier to classify the fault. The fault classifier makes comparison of the Markov parameters of faulty system with that of the nominal system and does the fault classification.

In section II a brief mathematical analysis of the proposed technique is dealt. Section II also addresses the proposed diagnosis approach. The proposed approach is illustrated on a standard benchmark problem and the simulation results are summarized in section III. Finally, we conclude by giving some comments on the application of the technique.

MATERIALS AND METHODS
The proposed method deals with identification of deviation in Markov parameter for fault in the plant and in the actuator. Markov parameters are the unique combination of input / output and system matrices. The approach consists of the following steps.

i. System identification
ii. Computation of Markov parameters
iii. Extraction of residues/error
iv. Fuzzy based fault classification

![Fig.1 Block diagram of the proposed approach.](image-url)
The identification process first constructs the input Henkel matrix $U_{kl}$ from the input data and the output Henkel matrix $Y_{kl}$ from the output data. The Hankel matrices are used to determine Oblique projection $O_i$ which is further decomposed using singular value decomposition (SVD). The system is identified using least square method. The Markov parameters are calculated for the identified system. Faults are introduced at random in the system and in the actuator. The Markov parameters of the faulty system are calculated and compared with the identified system. Fault residue is computed by finding the difference between the Markov parameters of the healthy and faulty system. The fault residues are given as input to a mamdani fuzzy inference system for classification.

**System identification**

System identification aims at constructing the mathematical model from the known inputs and output response of the system. As state space model is used in widespread in control theory, the objective is to identify the system matrices $A, B, C$ and $D$ of unknown system from the known input and output measurements [15]. The stochastic approach first defines the input Henkel matrix $U_{kl}$ from the input data and the output Henkel matrix $Y_{kl}$ from the output data. The oblique projections are then determined which is further decomposed using Singular Value Decomposition (SVD). SVD reduces computational load and noise sensitivity. The state sequences are formulated and solved in least square sense to obtain the system matrices $A, B, C$ and $D$.

**Markov parameter computation**

The impulse response terms $CA^{n-1}B$ for $n \geq 0$ (where $n$ is the order of the system) are known as Markov parameters or impulse response coefficients. Unlike eigen values, Markov parameters are unaffected by system transformation [4]. The specialty of Markov parameters is that it gives unbiased estimate of the system matrices with the state sequence approach. Given the values of system matrices $A, B$ and $C$, its impulse response can be constructed as Markov parameters $h_0, h_1, h_2, \ldots, h_n$.

**Residue generation**

The fault residue is the difference between the Markov parameters of the healthy and faulty system [6]. The known fault condition is simulated into the nominal system and the fault residue is extracted by comparing the Markov parameters of the healthy system and faulty system.

**Fuzzy classification**

In this work, a fuzzy classifier is implemented in order to process the residue for fault detection. The basic architecture of fuzzy system is shown in Fig.2. Fuzzifier converts the crisp input to a linguistic variable using the membership functions stored in the fuzzy knowledge base [5]. Inference Engine converts the fuzzy input to the fuzzy output using If-Then type fuzzy rules [13],[14]. The block diagram of Fuzzy Inference System (FIS) is shown Fig. 3.
Defuzzifier converts the fuzzy output of the inference engine to crisp using membership functions analogous to the ones used by the fuzzifier. Five commonly used defuzzifying methods are Centroid of Area (COA), Bisector of Area (BOA), Mean of Maximum (MOM), Smallest of Maximum (SOM), Largest of Maximum (LOM) are shown in Fig. 4. Mamdani method is widely accepted for capturing expert knowledge. It allows us to describe the expertise in more intuitive, more human-like manner. Mamdani fuzzy models are characterized by having fuzzy propositions as antecedents and consequences. The rule $l$ of the rule base for Mamdani fuzzy systems is IF $x_1$ is $A_{1l}$ AND ... AND $x_p$ is $A_{pl}$ THEN $y = y_l$; for each rule inference is calculated as $\mu_l(x_i) = \min[\mu_1l(x_i), \mu_21(x_i), \ldots, \mu_p1(x_i)]$. In this proposed algorithm max-min composition and centroid defuzzification are used as shown in Fig. 5.
i. Mathematical Analysis of the proposed technique
   
   a. Formation of Hankel matrix and oblique projections

   Consider the linear, fault free, stochastic discrete system,
   \[ x_{k+1} = Ax_k + Bu_k + w_k \]
   \[ y_k = Cx_k + Du_k + v_k \]

   where
   \( A \in \mathbb{R}^{n \times n} \) is the system matrix
   \( B \in \mathbb{R}^{n \times m} \) is the input matrix
C ∈ R^{m×n} is the output matrix

D ∈ R^{m×m} is the direct transmission matrix

v_k ∈ R^{m×1} is the measurement noise

w_k ∈ R^{m×1} is the process noise.

The problem is to determine the system matrices A, B, C and D up to within a similarity transformation given the large number of input and output data u_k and y_k, respectively.

Define the input Henkel matrix U_{k|1} from the input data as,

\[
U_{k|1} = \begin{bmatrix}
U_k & U_{k-1} & \cdots & U_{k-j+1} \\
U_{k+j} & U_{k+j-1} & \cdots & U_{k+1}
\end{bmatrix}
\] (2)

Similarly, define the output Henkel matrix Y_{k|1} from the output data,

\[
Y_{k|1} = \begin{bmatrix}
Y_k & Y_{k-1} & \cdots & Y_{k-j+1} \\
Y_{k+j} & Y_{k+j-1} & \cdots & Y_{k+1}
\end{bmatrix}
\] (3)

The matrix containing the past inputs U_p and outputs Y_p is W_p given by,

\[
W_p = \begin{bmatrix}
Y_p \\
U_p
\end{bmatrix}
\] (4)

The oblique projection is determined using

\[
O_i = Y_{i|2i-1} / U_{i|2i-1} W_{o|i-1}
\] (5)

\[
O_{i+1} = Y_{i+1|2i-1} / U_{i+1|2i-1} W_{o|i}
\]

It is assumed that the input u_k is uncorrelated with the noise w_k and v_k, the number of available measurements is sufficiently large (i.e. j → ∞) and the noise w_k and v_k are non-identically zero.

b. **Singular value decomposition**

The oblique projection formed is decomposed using singular value decomposition and the following result is obtained.

\[
O_i = U S V^T_i
\]

\[
= \begin{pmatrix}
U_1 \\
U_2
\end{pmatrix}
\begin{bmatrix}
S_1 & 0 \\
0 & S_2
\end{bmatrix}
\begin{pmatrix}
V_1^T \\
V_2^T
\end{pmatrix}
\] (6)

The order n of the system is equal to the order of S_1.

c. **Identification**

Determine the state sequences:

\[
\hat{x}_i = S_1 V_i^T
\]

The matrices $A, B, C, D$ are then determined by solving the least-squares problem:

$$\min_{A,B,C,D} \left\| \begin{pmatrix} \bar{X}_{i+1} \\ \bar{Y}_{i+1} \end{pmatrix} - \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \bar{X}_i \\ \bar{Y}_i \end{pmatrix} \right\|$$

(7)

In the identification parlance, usually the Eigen values or the Markov parameters of the original system and those of the identified system are compared as an index for identification.

d. **Computing the Markov parameters**

The impulse response of the state model can be easily computed by direct calculation. Let $x(0) = 0$. Then,

$$h(0) = Cx(0)B + D = D$$

$$h(1) = Cx(1) = CB$$

$$h(2) = Cx(2) = CAB$$

$$h(3) = Cx(3) = CA^2B$$

$$h(n) = C A^{n-1} B, n > 0$$

(8)

The impulse response coefficients, also known as Markov parameters of the state model can be summarized as

$$h(n) = \begin{cases} D, & n = 0 \\ C A^n - 1B, & n > 0 \end{cases}$$

(9)

If Markov parameters of the original and identified system are the same, then the identified system and actual system are the same. In this manner, the Markov parameters serve as an index for the identification algorithm.

e. **Mathematical analysis for the faulty system**

i. **System Fault**

Consider the linear faulty system,

$$x_{k+1} = (A + \Delta A)x_k + Bu_k + w_k$$

$$y_k = Cx_k + Du_k + v_k$$

where $\Delta A \in \mathbb{R}^{n \times n}$ is the system fault.

The Markov parameters for the identified faulty system is computed as follows:

$$M_i(n) = C (A + \Delta A)^{-1} B, n=1,2 \ldots$$

(11)

ii. **Actuator Fault**

Consider the linear faulty system,
\[ x_{k+1} = Ax_k + (B + AB) u_k + w_k \]  
\[ y_k = Cx_k + Du_k + v_k \]

where \( AB \in \mathbb{R}^{nm} \) is the actuator fault.

The Markov parameters for the identified faulty system is computed as follows:

\[ M_d(n) = C A^{n-1} (B + AB), \quad n = 1, 2 \ldots \]  

\[ e = M(n) - M_d(n). \]  

**RESULTS**

The proposed fault diagnosis approach is applied to a linear system and the results are presented in this section. The system is identified with the given input and output measurement values. When faults are introduced at random instances, Markov parameters of the system vary from its nominal value. This variation is applied to a fuzzy system to classify faults.

Consider the system given by

\[ x_{k+1} = ax_k + bu_k + w_k \]  
\[ y_k = cx_k + du_k + v_k \]

\[ a = [0.603 0.603 0 0; -0.603 0.603 0 0; 0 0 -0.603 -0.603; 0 0 0.603 -0.603]; \]
\[ b = [1.1650, -0.6965; 0.6268, 1.6961; 0.0751, 0.0591; 0.3516, 1.7971]; \]
\[ c = [0.2641, -1.4462, 1.2460, 0.5774; 0.8717, -0.7012; 0.3300, -0.6390; 0.3600]; \]
\[ d = [-0.1356, -1.2704; -1.3493, 0.9846]; \]

The eigen values of the given system are \( 0.6030 + 0.6030i, 0.6030 - 0.6030i, -0.6030 + 0.6030i, -0.6030 - 0.6030i \). The eigen values lie within the unit circle and hence the system is stable. The inputs are chosen as random sequence and the corresponding outputs are generated.

The identified system is as follows:

\[ x_{k+1} = Ax_k + Bu_k + w_k \]  
\[ y_k = Cx_k + Du_k + v_k \]

\[ A = [0.4815, 0.5753, 0.2624, 0.1805; -0.6473, 0.4235, 0.4902, -0.0297; 0.1718, 0.5013, -0.3941, -0.6211; 0.1992, -0.1368, 0.5975, -0.5181]; \]
\[ B = [-0.0161, 1.8717; -0.9258, 0.2399; -0.7152, -1.0181; -0.0489, -0.3583]; \]
\[ C = [-1.4577, 1.1081, -1.0238, -0.1295; -0.7152, -0.6432, 0.3332, -0.4639]; \]
\[ D = [-0.1490, -1.2905; -1.3556, 0.9805]; \]
The eigen values of the identified system are 0.6024+0.6042i, 0.6024-0.6042i, -0.6040+0.6031i, -0.6040-0.6031i. The eigen values of the identified system closely match with the actual system. The system identification process is validated by comparing Markov parameters as this approach directly involves the system matrices. In this work first 20 Markov parameters are used for comparison purpose and are tabulated in Table 1.

Table I: First 20 Markov Parameters of the original and identified system

<table>
<thead>
<tr>
<th>First 20 Markov parameters</th>
<th>Markov parameters of actual system</th>
<th>Markov parameters of identified system</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.9757</td>
<td>3.9677</td>
</tr>
<tr>
<td>2</td>
<td>2.1260</td>
<td>2.1329</td>
</tr>
<tr>
<td>3</td>
<td>1.6619</td>
<td>1.6531</td>
</tr>
<tr>
<td>4</td>
<td>1.5435</td>
<td>1.5509</td>
</tr>
<tr>
<td>5</td>
<td>2.1025</td>
<td>2.1066</td>
</tr>
<tr>
<td>6</td>
<td>1.1243</td>
<td>1.1326</td>
</tr>
<tr>
<td>7</td>
<td>0.8789</td>
<td>0.8698</td>
</tr>
<tr>
<td>8</td>
<td>0.8163</td>
<td>0.8188</td>
</tr>
<tr>
<td>9</td>
<td>1.1119</td>
<td>1.1184</td>
</tr>
<tr>
<td>10</td>
<td>0.5946</td>
<td>0.6016</td>
</tr>
<tr>
<td>11</td>
<td>0.4648</td>
<td>0.4577</td>
</tr>
<tr>
<td>12</td>
<td>0.4317</td>
<td>0.4323</td>
</tr>
<tr>
<td>13</td>
<td>0.5880</td>
<td>0.5938</td>
</tr>
<tr>
<td>14</td>
<td>0.3144</td>
<td>0.3196</td>
</tr>
<tr>
<td>15</td>
<td>0.2458</td>
<td>0.2409</td>
</tr>
<tr>
<td>16</td>
<td>0.2283</td>
<td>0.2283</td>
</tr>
<tr>
<td>17</td>
<td>0.3110</td>
<td>0.3152</td>
</tr>
<tr>
<td>18</td>
<td>0.1663</td>
<td>0.1699</td>
</tr>
<tr>
<td>19</td>
<td>0.1300</td>
<td>0.1269</td>
</tr>
<tr>
<td>20</td>
<td>0.1207</td>
<td>0.1205</td>
</tr>
</tbody>
</table>
The faulty matrix of known magnitude is introduced at arbitrary time instants. The faults are introduced in the system matrix as follows:

\[ \Delta A = \begin{cases} 0; 0 \leq t \leq 10 \\ 0.5; 11 \leq t \leq 15 \\ 0; 16 \leq t \leq 25 \\ 0.8; 26 \leq t \leq 35 \\ 0; 35 \leq t \leq 40 \end{cases} \]  

(19)

The first 20 Markov parameters of faulty system are extracted and compared with the Markov parameters of the nominal system and the plots obtained are as shown in fig. 6.

The faults are introduced in the actuator as follows:

\[ \Delta B = \begin{cases} 0; 0 \leq t \leq 10 \\ 0.5; 11 \leq t \leq 15 \\ 0; 16 \leq t \leq 25 \\ 0.8; 26 \leq t \leq 35 \\ 0; 35 \leq t \leq 40 \end{cases} \]  

(20)

Fig. 6 Markov parameters of identified system and faulty system, Error in Markov parameters due to faults
The first 20 Markov parameters of faulty system (with actuator fault) are extracted and compared with the Markov parameters of the nominal system and the plots obtained are as shown in fig. 7. The fuzzy classifier is used to classify faults. Mamdani fuzzy inference system constructed for fault identification is shown in Fig.8.

Fig. 7 Markov parameters of identified system and (actuator fault) faulty system, Error in Markov parameters due to Actuator faults

Fig. 8 Single input single output Mamdani fuzzy inference system
The membership functions defined for input and output variables and the rules constructed are given in Lemma 1.

**Lemma 1**: The fuzzy rules are

- If (input1 is imf1) then (output1 is omf1)
- If (input1 is imf2) then (output1 is omf2)

where ‘imfx’ and ‘omfx’ are input membership function and output membership function of fault index ‘x’ respectively. The output of FIS for the given input is shown in Fig.9 and Fig. 10. If the residue value lies within the specified limit, the system is considered to be fault free and the output of the fuzzy classifier is set as 0. If the residue value exceeds the given limit, the system is said to be faulty and is indicated with the classifier output 1.

![Fig. 9.a. Fuzzy classification for system fault (When no fault is present, output = 0)](image1)

![Fig. 9.b. Fuzzy classification for system fault (With the presence of fault, output = 1)](image2)
DISCUSSION
In this work, a new approach to fault diagnosis using Markov parameters and classification based on Fuzzy classifier has been presented. The significant advantage of this approach is that it gives unbiased estimates of the parameter variations in a straightforward way. Hence it minimizes the effects of ill conditioning of input signals and external disturbances. The fuzzy classifier have been successfully applied to classify the random faults applied to the system at different time instances. Interestingly, the task is accomplished without having to compute explicitly the system dependent interaction matrix itself. The main aspect of this work was the use of linear system fault diagnosis to avoid the complexities that would otherwise be inevitable if nonlinear models are used. Though
there is an increasing interest in the research literature in the use of non-linear methods, it is only a question of time before these techniques find their way into full application projects. However, as the feature of system supervision is to monitor the operation and performance of the system with respect to an expected point of operation, linear system methods are still very valid. Deviations from expected system behavior could be used to monitor system performance changes as well as system component malfunctions. One drawback of the proposed method is that it has a relatively complicated procedure. The main contribution of this paper is the demonstration of a Markov parameter based fault detection and diagnosis of the linear system. The proposed algorithms have been validated by means of valid case study. Owing to the faster identification the work could be easily employed in an industrial environment for detecting the deterioration of parameters that age with time automatically.

CONCLUSION

A new approach towards the fault diagnosis of linear dynamical systems using Markov parameters is proposed in this paper. The given system is excited using random input signals and corresponding output signals are generated. Using the input – output measurements, the system identification is done using the mathematical approach. Markov parameters are used as a performance index for classifying healthy and faulty systems. The fuzzy classifier is used to classify faults. The future work is oriented towards developing the fault tolerant control.

REFERENCES


