FORWARD KINEMATICS AND DIMENSIONAL SYNTHESIS OF TRIPOD AND TRIGLIDE PARALLEL MANIPULATOR

Arockia Selvakumar.A\textsuperscript{1}, R.Sivaramakrishnan\textsuperscript{2} and K.Kalaichelvan\textsuperscript{3}

ABSTRACT
Two 3 - RPS Parallel Manipulator is proposed in this paper. The aim of this paper is to design suitable parallel manipulator for angular drilling applications. The proposed two 3-RPS parallel manipulator have two orientation freedom and one translation freedom. The forward kinematic problems and synthesis are carried out by considering geometrical parameters of the mechanism. First, the dimensional synthesis is carried out to get maximum angle of tilt of the moving platform by kinematic equations by simple approach. Secondly, the parallel manipulators are modeled and simulated by using ADAMS® Package. The simulated results are compared with the derived kinematic equations. The simulation results are matched with the derived kinematic equations with minimal error. From this study, the selection of the parallel manipulator for angular machining application is given in detail.

1. INTRODUCTION
Parallel manipulators have some significant advantages like more rigidity and accuracy, higher force and torque capacity, and simpler inverse kinematics over conventional serial manipulator. In general it can be expected that the robot manipulator having the end effectors connected to the ground via- several chains having actuation in parallel will have greater rigidity and superior positioning capability. Because of the recent trend towards high speed machining, there is a demand to develop machine tools with high dynamic performance, improved stiffness and reduced moving mass [1]. Several tripod units proposed in the literature have attracted attention from industry. The most successful one is Tricept from Neos Robotics AB, which is mainly designed for heavy duty cutting applications. Georg V and Parawrist are both designed Tricept with fixed-length actuators for light duty applications [2]. A spatial 3-RPS parallel manipulator with three identical limbs, developed by Lee and Shah, is one such manipulator with 3 – DOF [3]. Joshi and Tsai determined two singular positions of the 3- RPS manipulator from the jacobian, determined by making use of theory of reciprocal screws [4]. Merlet presented an algorithm to determine all the possible geometries of Gough type 6-DOF parallel desired workspace, taking in to account the leg length limits, mechanical limits on the passive joints and interface between links [5]. Q.Y. Wang, H.Zou mentioned that the traditional architectural model of the machine tool is facing more and more challenges. The concept of flexibility is considered to be the most important factor with respect to the classical system [6]. Simulation has been recognized as an important research tool since the beginning of the 20\textsuperscript{th} century. However, the “good times” for simulation started with the development of computers and now the simulation is a powerful visualization, planning and strategic tool in different areas of research and development [7]. This paper illustrates the mechanism of Tripod and Triglide
type of parallel manipulator for angular machining applications. The concept of parallel manipulator is becoming more popular in modern manufacturing processes due to its various inherent advantages like rigidity, less inertia and precise positioning [8]. In the Triglide type of parallel manipulator the lead screws were kept in a horizontal manner. But in the Tripod model the lead screw is kept in a vertical manner. Each link is controlled by actuator which can be mounted at or near the fixed base. The entire manipulator system is made computer controlled for making the system more flexible, accurate and user friendly. The movable platform of the parallel manipulator can be tilted to required angle by giving suitable pulses (input) to the stepper motors which actuates the links connecting the movable platform. In this design each and every component (Lead screws, Bushes, Circular movable platform. In this design each and every motors which actuates the links connecting the giving suitable pulses (input) to the stepper manipulator can be tilted to required angle by system is made computer controlled for making the mobile can be manipulated and the entire manipulator payload of 7Kg and is fabricated by avoiding the system is made computer controlled for making the system more flexible, accurate and user redundant motion.

2. Parallel Mechanism
The mechanism of the model parallel manipulator basically consists of a platform called mobile which houses the cutting tool, 3 rigid links and a fixed platform called the base [9]. The mobile is connected to the links by means of Spherical joints. The other ends of the links are connected base through pin joints. With the link lengths changing, the mobile can be manipulated and the tool changes its position and orientation with respect to the base. In this paper symmetrical type of parallel manipulator is considered. In a symmetrical parallel manipulator, the Number of links (N) is equal to the number of degrees of freedom (F), Which is also equal to the total number of loops, L+1 (Including external Loop); that is shown in equation (1)

\[ N = F = L + 1 \]  

**...(1)**

Fig. 1. Geometrical model of 3 - RPS Parallel Manipulator

2.1. Mobility equation
The considered mechanism is three degrees of freedom parallel manipulator having a mobile platform connected to a fixed platform by three rigid links by means of ball joints and pin joints. The three tripods are designed using three motion axes to achieve three degrees of freedom. This can be explained by the following mobility equation,

\[ F = d(n - g - 1) + \sum f_i \]  

\[ \text{......... (2)} \]

Where,

F-Mobility or the degrees of freedom of Mechanism,  
d -Order of the system,  
n - the number of bodies in mechanism,  
g - Number of joints and  
f_i - Number of degree of freedom of the i-th joints.

For the presented mechanism n=8, g = 9 and f_i = 3 for each spherical joint and f_i=1 for each revolute joints and f_i=1 for screw joint. Application of equation yields,

\[ F = 6 (8-9-1) + 3 (3+1+1) = 3 \]

The 3 DOF of the mechanism are, the rotation about X axis, rotation about Y axis and translation along Z axis [10].
3. Kinematic analysis of Parallel manipulator

The kinematics considers the motion conversion in the spherical joints and the pin joints. The ball joints which has three rotational DOF is used to rotate the moving platform in any direction and the pin joint which connects the lower end of link has one DOF. The forward kinematics refers to the computation of the position or motion of each link as a function of joint variables. The kinematic equation for finding the angle of moving platform is to be found in terms of link length, joint angles and radius of platforms. The position analysis of single link movement is shown in Fig 2 and 3.

3.1 Fixing the coordinate system

A base Cartesian coordinate system X, Y and Z (Global Coordinate System) is fixed at the center of the base platform with Z axis pointing vertically upward and the X axis pointing towards the pin joint 1 (p1) of the first link 1. Similarly a coordinate system (local coordinate system) x, y and z is assigned to the center of the upper platform with the z axis normal to the platform and x axis pointing towards the spherical joint S1.

3.2 Cartesian Position of Moving Platform

Since the ball joints are placed at vertices of an equilateral triangle, the Cartesian position or the origin of the X, Y, and Z frame is essentially the centroid of the triangle [11]. From figure 2, Forward kinematics of single link movement of Tripod parallel manipulator can be formulated from the equation (3) to (10),

\[
\begin{align*}
L_1 &= 1.5x \\
X &= L_2 + L \cos \theta_2 \\
D &= P \times n \\
\varphi &= H - \theta \\
\cos \theta_2 &= \frac{L^2 + X^2 + H^2 - D^2}{2Lx} \\
\cos \varphi &= \frac{L^2 + X^2 + L_1^2 - L^2}{2Lx} \\
\theta &= \theta_3 - \theta_1
\end{align*}
\]

Fig. 2. Single Link Movement of Tripod
From figure 3, Forward kinematics of single link movement of Triglide parallel manipulator can be formulated from the equation (3) to (6) and (12) to (13),

\[
\cos \theta_2 = \left\{ \frac{(X-D)^2 + H^2 + L_1^2 - L^2}{2 \cdot L_1 \sqrt{((H + (X-D))^2)}} \right\}
\]

(12)

\[
\theta = \theta_1 - \tan^{-1}(H / (X - D))
\]

(13)

4. Kinematic synthesis of parallel manipulator

Synthesis of mechanism is the design or criteria of mechanism to produce a desired output motion for a given input motion or in other proportions of mechanism for the given and output motions. The dimensional synthesis is taken into account for the determination of suitable dimensions of mechanism by logical approach. The link lengths were considered to have 200, 300, 400 and 500 mm. Similarly, the dimension of the radius of the moving platform is considered to have 50, 60, 70, 80, 90 and 100 mm and the simulation of parallel manipulator is done by using the ADAMS package and based on the simulation results synthesis of the mechanism carried out.

5. Kinematic Modelling and Simulation of parallel manipulator

The parallel manipulator is modeled by considering a payload of 10 kg with the factor of safety 3. The maximum angle of tilt of each of the axes is considered to be 65 degrees (for the displacement of nut 50 mm) at any of the axes. Considering these specifications as target parameters, the parallel manipulators are modeled and simulated. The various parts that are modeled are lead screws, links, spherical joints, (ball & socket joint) ball bearings, base plate etc. The links made of mild steel rods of lengths 300 and 500 mm with a diameter of 15 mm is used by considering the compressive stress acting on them. Similarly the lead screw is also designed to withstand the force acting on it. The ball and socket joint was selected to to withstand the load
and the compactness for light weight. Proper bearings were provided for support by considering the friction and the force acting in them. The torque was calculated and for that torque, the stepper motor was selected with a step angle of 1.8°.

5.1. Working Principle of the manipulator
In order to achieve the angle of mobile platform, the motor, which is connected to the leg, should be actuated. This is achieved by giving a correct number of pulses to the motor according to the given angle of inclination. Once the output variable (task) is entered, the input variable is given in terms of number of pulses. A software program is written in such a way that the 3 motors or a single motor can be actuated at a time simultaneously or separately depending upon the angular tilt required at the platform. The tilt can be in either with respect to X or Y axes or it can be a combination of X & Y or Y & Z or X & Z directions and the simulated models are shown in fig 4 & 5.
5.2 Simulation
The manipulators model have been constructed in the ADAMS by building the physical attributes of the parts in the mechanical systems that has rigid bodies, point masses, flexible bodies and constraints.

The simulation of the mechanisms are carried out by considering two links are kept constant and one link is allowed to move in the Z – direction to find the maximum rotation of the moving platform about Y – axis. Similarly, the simulations are carried out in other possible ways.

6. Results and Discussion
The kinematic analysis and synthesis of the 3 - DOF parallel manipulators have been carried out and the results are verified by analytical approach and Simulation package. The results are given in Tables 1 and Figure 6. In Table 1, the simulation results are compared with the derived kinematic equations and it shows that the Triglide mobile (Moving platform) has tilted to an angle of 25.26° to 17.76° and 25.63° to 18° about Y-axis by the actuation of one link (for the link lengths of 200 to 500 mm). Similarly, for Tripod it is found to be 36.20° to 36.08° and 36.41° to 36.33°. By comparing the results the maximum error obtained is 2.5% by the ADAMS simulation.

<table>
<thead>
<tr>
<th>Link Length in mm</th>
<th>Angular tilt of mobile platform in °</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ADAMS</td>
</tr>
<tr>
<td>200</td>
<td>25.26</td>
</tr>
<tr>
<td>300</td>
<td>20.48</td>
</tr>
<tr>
<td>400</td>
<td>18.46</td>
</tr>
<tr>
<td>500</td>
<td>17.76</td>
</tr>
</tbody>
</table>

Figure 6 Comparision between ADAMS and Analytical Method
Fig. 7. Analytical results for Link Length 200 mm

Fig. 8. Analytical results for Link length 300 mm

Fig. 9. Analytical results for Link length 400mm
Fig. 10. Analytical results for Link length 500 mm

Fig. 11. Analytical results for mobile radius 100 mm

Fig. 12. Analytical results for mobile radius 90 mm
Fig. 13. Analytical results for mobile radius 80 mm

Fig. 14. Analytical results for mobile radius 70 mm

Fig. 15. Analytical results for mobile radius 60 mm
In the above mentioned cases the error obtained in ADAMS package is due to arresting the motion of other two links while simulating the model. Also the error may be due to fixing the exact coordinates in the working grids of ADAMS package. Since values obtained from analytical results are matching with ADAMS results, the values obtained from them are considered to be the correct value.

Analytical studies were carried out by comparing the parameters of parallel manipulator (refer fig 11 and fig 16), the results shows that when the mobile radius is increased from 50 mm to 100 mm for constant link lengths of 200 the angle of tilt of the mobile decreases from 25.63° to 11.09° for Triglide and 36.05° to 18.09° for Tripod parallel manipulator but when the link length is increased from 200 to 500 mm for constant mobile radius of 50 mm the angle of tilt of mobile increases from 36.05° to 36.33° in Tripod whereas in Triglide it decreases from 26.53° to 18°. The results are similar from fig 13 to 15 for the link lengths 300 to 500 and fig 16 to fig 20 for the mobile radius 60 to 100 mm.

7. CONCLUSION

Two 3 - DOF parallel manipulators have been modeled and analyzed for angular machining applications. These types of manipulator can be used in place of machine tool table for machining purposes like drilling angular and inclined holes on components and contour milling using end milling cutters for the maximum angle of tilt of mobile platform. The following conclusions are made based on the analytical method and Simulation results, First, Single link kinematic analysis carried out for links actuation individually shows that the Simulation values are closer with the analytical values. Secondly, the accuracy of movement of platform is found to be better since the values of ADAMS matching with the analytical and finally, Analytical studies results were proved that when the mobile radius is increased for constant link length the angle of tilt of the mobile decreases for both Triglide and Tripod parallel manipulator but when the link
length is increased for constant mobile radius the angle of tilt of mobile increases in Tripod whereas in Triglide it decreases. The selection of the manipulator is depends on the designer choice. By using ADAMS Package, Velocity and acceleration analysis and calculation of forces and torques of the parallel manipulators can be studied for prototype.

REFERENCES
