ABSTRACT
Water management is the topic duly been recognized as a priority sector in the recent days. Flow measurement has rightly acknowledged as the most important part of water management to judicious usage of all important resource water. Sharp crested weirs are relatively accurate flow measuring devices and are most commonly used devices in Open channels. Lot of research has gone through in case of flow over different shapes of normal weirs, side weirs and oblique weirs. Although sharp crested weirs are more commonly used in laboratory, they are seldom used in the field channels, as their major drawback is afflux development resulting in Large Head loss (Coefficient of Discharge is a measure of head loss). To improve upon these drawbacks Shesha Prakash and Shivapur worked on inclined weirs. They have developed general head-discharge equation through the model can provide a reasonably accurate method to predict flow over inclined triangular notch. The considerable increase in the discharging capacity of the weir, with increase in inclination along the vertical plane of weir is ascertained. The established discharge-head-inclination equation through the model can provide a reasonably accurate method to predict flow over inclined trapezoidal notch. The higher discharge coefficient associated with angle $\alpha$ will help to improve discharge capacity and reduce afflux.

KEY WORDS
Sharp-crested Weirs; Inclined Notches, Trapezoidal Notch, Discharge characteristics, Flow measurement, Afflux.

1. INTRODUCTION:
A number investigation on weir profiles have been done by many researchers for flow measurements mainly to reduce the afflux, increasing the discharging capacity and practically usable and acceptable weir profile. Normally the weirs are fixed normal to the flow axis and lot of experimental work is available through the past literature. However, these weirs can also be arranged oblique to flow axis known as oblique weir and side weirs for some gains in flow measurements. Further, any weir is associated with weir constant, called discharge coefficient which is a measure of its discharging capacity. De Marchi (Henderson 1966), Frazer (Cheong 1991), Subramanya and Awasthy (1972), Nadasamoorthy and Thomson (1972), Ranga Raju et al (1972), Hager (1987), Cheong (1991), Singh, et al (1994) have analysed flow over rectangular side weir. Aichel (1953) gave table relating the discharge coefficient of a round crested skew weir to that of a normal weir. Ganapathy et al (1964) conducted experiments on broad crested skew weir and gave curves for discharge coefficient and head with skew angle ‘0’ as the third parameter. Mahapatra (1965) followed approach of Aichel for rectangular sharp crested weirs. Muralidhar (1965) conducted experiments on weirs of finite crest width. Swamee et al (1994) developed equation for elementary discharge coefficient for rectangular side weir. Jain and Fischer (1982) have studied weirs with crest oblique to the flow axis in the channel. Multi fold skew weirs called Labyrinth weirs were developed and tested by Hay and Taylor (1970). They presented results in the form of curves between discharge coefficient and weir head to weir crest ratio and compared the results with normal weir. Tullis et al (1995) studied Labyrinth weir having trapezoidal plan forms and presented results in the form of curves between discharge coefficient and E/w with angle ‘0’ as the third parameter. Ramamurthy and Vo Ngoc-Diep (1994) have shown that with the increase in downstream slope for circular crested weir discharge coefficient can be improved. Talib Mansoor (2001) has developed equation for elementary discharge coefficient for rectangular skew weirs. Shesha Prakash and Shivapur (2002) have studied the variation of discharge coefficient with the angle of inclination of rectangular weir plane with respect to the normal (standard) position of plane of weir for a sharp crested trapezoidal weir. Most of the researchers have reported their study on establishing discharge coefficient in terms of weir head to weir (height/length) and skew angle. The majority of these investigators have used other type of sharp crested weirs to calibrate the weir which is under investigation, limiting the accuracy of the results obtained. Shesha Prakash et. Al. have developed a head-discharge-inclination model for flow through inclined rectangular notch (2010) and inclined trapezoidal notch (2010).
In the present investigation, sharp-crested Trapezoidal weir of 100 mm crest length and side slope 1:8 is fixed inclined with respect to the bed of the channel. The conventional method of volumetric measurement is used to find actual discharge over the weir.
As can be seen from Fig. 1, the flow length along the plane of the weir increases with increase in inclination with the normal plane which increases the discharging capacity of the weir.

2. EXPERIMENTS:
Experiments were carried on inclined Trapezoidal notch fixed normal to the flow direction (0°), 15°, 30°, 45°, and 60° inclinations with respect to the normal plane (Vertical) along the flow axis. The experimental channel is rectangular in section and having dimensions 0.28m wide, 0.45m deep and of 11.5m length. The channel is constructed of Perspex sheet and has smooth walls and bed to reduce the boundary frictional force with nearly horizontal bed. It is connected to a Head tank of dimensions 1.5mx1.2mx1.5m. The Trapezoidal notch is made of 8mm Flexy glass with a crest thickness of 1 mm and a 45° chamfer given on downstream side to get a springing nappe. The experimental set up is shown in Fig. 2. Water is supplied to the channel by an inlet valve provided on supply pipe. Over head tank is provided with overflow arrangement to maintain constant head. Smooth, undisturbed, steady-uniform flow was obtained by making the water to flow through graded aggregates and the surface waves were dampened by tying gunny bags at the surface near the tank. The head over the weir is measured using an electronic point gauge placed in piezometer located at a distance of about 1.40m on upstream of inclined trapezoidal notch. A collecting tank of size 1.465m length, 1.495m breadth, and of 1.5m depth is provided with a piezometer. Water after running through the experimental setup is collected in an underground sump from which it is re-circulated by pump by lifting it back to the overhead tank. In the present study, the conventional method of volumetric discharge measurement is used, which increases the accuracy of the work. The measurements are done through electronic gauge which automatically detects the water level and records the gauge reading. The volumetric measurement is done through self regulated timer for a fixed rise of water level automated through sensors.

3. PROCEDURE:
The step by step procedure followed for experimentation is as follows.
1. A sharp crested trapezoidal notch was installed at a desired inclination ‘α’ and height ‘z’.
2. The crest reading was taken by electronic gauge when the water was in verge of flowing past the weir crest.
3. The discharge in the channel was controlled by the valve provided on supply pipe.
4. When flow attained a steady-uniform condition, the head over the weir crest ‘h’ was measured through the electronic point gauge.
5. For 100 mm, 200 mm, 300 mm and 400 mm rise of water level in collecting tank time in seconds was recorded. (To eliminate the human error in piezometric readings in the collecting tank, time was auto-recorded by an electronic timer, for the above interval of water rise in the tank and averaged, by considering the cumulative volume and the accumulated time).
6. After having varied the discharge uniformly with a near constant increments in channel using a control gate of supply pipe, the steps 4 to 5 was repeated.
7. Steps from 1 to 6 were repeated for new desired angle of inclination of notch plane ‘α’.

The present investigation was carried out on the range of variables shown in Table1.

<table>
<thead>
<tr>
<th>Position of the weir</th>
<th>Normal</th>
<th>Angle of inclination in degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>α=0°</td>
<td>15°</td>
</tr>
<tr>
<td>Actual discharge (m³/s)</td>
<td>0.01455 to</td>
<td>0.01617</td>
</tr>
<tr>
<td>Head over the crest (m)</td>
<td>0.26000</td>
<td>0.25480</td>
</tr>
<tr>
<td>No. of runs</td>
<td>45</td>
<td>38</td>
</tr>
</tbody>
</table>

Table 1. Range of variables studied

As can be seen from Fig. 1, the flow length along the plane of the weir increases with increase in inclination with the normal plane which increases the discharging capacity of the weir.
4. ANALYSIS OF RESULTS:
Initially the obtained and computed values of Head and discharge were non-dimensionalised as below so that the obtained equation will be more generic in nature. The head is non-dimensionalised is done with respect to width of the channel to include the influence of reduction in flow cross-sectional area due to installation of the weir.

The discharge for flow through trapezoidal notch given by Darcy-Weisbach equation is

\[ q = \frac{2}{3} \sqrt{2g \left( \frac{2}{3} Lh^2 + \tan \theta h^3 \right)} \]

Non-dimensionalising the above equation, we get

\[ Q = \left[ \frac{3}{5} H^\frac{3}{2} + \frac{4}{5} \left( \frac{B-1}{2} \right) H^\frac{1}{2} \right] \]

Where \( q \) is the discharge through the weir in m³/s, \( L \) is the crest length, \( \alpha \) is the angle of the side slope with vertical, \( b \) is the Channel width in m and \( h \) is the head over the crest in m.

Non-dimensionalising the above equation, we get

\[ Q = \frac{q}{2 \sqrt{2g L h_i^2}} \quad ; \quad H = \frac{h}{h_i} \quad \text{and} \quad B = \frac{b}{L} \]

![Fig. 2 Experimental setup](image)

![Fig. 3. Non-dimensional Head-discharge plot for various values of inclination \( \alpha \).](image)
A plot of Non-dimensional discharge verses non-dimensionalised head for various positions of plane of trapezoidal notch weir have been shown in Fig.3. It shows that the discharge increases with increase in inclination angle $\alpha$.

Hence, sharp-crested trapezoidal weir can be installed at a suitable inclination angle to the bed of the channel without any alteration to the conventional simple geometry of the weir so that the discharging capacity of the weir can be much higher, corresponding to the same head, as compared to conventional normal weir, which is evident from Fig. 3. This will help in reducing free board requirement on upstream of weir position. Further, it can also be used in the existing channels with least effect of afflux.

5. MATHEMATICAL MODELING:
This paper proposes a computerizable iterative algorithm to intermittently improve the efficiency of the interpolation by the well-known simple and popular Newton-Gregory Forward Interpolation Formula, using statistical perspective of Reduced-Bias. The impugned formula uses the values of the simple forward differences using values of the unknown function $f(x)$ at equidistant-points/ knots in the Interpolation-Interval say $[x_0, x_n]$. The basic perspective motivating this iterative algorithm is the fuller use of the information available in terms of these values of the unknown function $f(x)$ at the $(n+1)$ equidistant-points/knots. This information is used to reduce the Interpolation Error, which is statistically equivalent to the well-known concept of bias. The potential of the improvement of the interpolation is tried to be brought forth per an empirical study for which the function is assumed to be known in the sense of the simulating nature of the empirical study) and the interpolated values at the mid-points of the equidistant-points/ knots in Interpolation-Interval, (say $[0,1]$). This leads to the calibrations of the respective Percentage Relative (Relative to actual value of the function at that point) Errors and hence that of the respective Percentage Relative Gains in terms of the reduced values of the Percentage Relative Error, compared to that with the use of the Newton-Gregory original forward difference formula. Newton’s forward difference formula is significant in many contexts. It is a finite difference identity capable of giving an interpolated value between the tabulated points $\{f_k\}$ in terms of the first value $f_0$ and powers of the forward difference “$\Delta$”.

For $k \in [0,1]$; the formula states

$$ f_k = f_0 + k\Delta_1 + \frac{k(k-1)}{2!} \Delta_2 + \frac{k(k-1)(k-2)}{3!} \Delta_3 + \ldots + \frac{k(k-1)(k-2)\ldots(k-n+1)}{n!} \Delta_n $$

Where, $k$ is any real number; $\Delta_1, \Delta_2, \Delta_3, \ldots \ldots \ldots \Delta_n$ are respectively the first, second, third,………..nth forward differences.

This formula looks apparently like an analog of a Taylor Series expansion. It would be important to note that we have taken the Interpolation-Interval as $[0,1]$ rather than say $[a, b]$ without any loss of generality. In fact, $C[0,1]$ and $C[a, b]$ are essentially identical, for all practical purposes, in as much as they are linearly isometric as normed spaces, order isomorphic as lattices, and isomorphic as algebras (rings). The following section details the algorithm enabling the fuller use of the “information” available in terms of these values of the unknown function $f(x)$ at the $(n+1)$ equidistant-points/knots.

As the same perspective (of Reduced Error/Bias) is available to be used at the end of a particular iteration, beginning from the first iteration, the proposed algorithm is an iterative one. The discharge-head-inclination equation can be expressed as

$$ Q = f(\theta)H^{\phi(\alpha)} \quad (01) $$

The modeling part is subdivided into two stages.

In the first stage the actual head-discharge data values are statistically fit with the exponential equation

$$ Q = KH^\alpha $$

### Table 2 Calibrated Head-discharge equations for various angles $\alpha$

<table>
<thead>
<tr>
<th>S.No</th>
<th>$\alpha$</th>
<th>$Q=KH^\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>$Q=0.879 H^{1.628}$</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>$Q=0.893 H^{1.605}$</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>$Q=1.230 H^{1.696}$</td>
</tr>
<tr>
<td>4</td>
<td>45</td>
<td>$Q=1.354 H^{1.627}$</td>
</tr>
<tr>
<td>5</td>
<td>60</td>
<td>$Q=2.163 H^{1.670}$</td>
</tr>
</tbody>
</table>

Fig. 4 Variation of weir coefficient ($K$), Head indices ($n$) and relative discharge coefficient $C_{di}$ with weir inclination ($\alpha$)

In the second stage adopting the Newton-Gregory Forward Interpolation Formula to develop the model for the present problem, we see that there are 5 values and hence we can adopt only value up to n-1, i.e. 4th order polynomial curve can be fit to the data and simplifying the equations, we get the final general head-discharge-angle expression for any given notch and inclination as under:

$$ Q = KH^\alpha $$
\[ K = \left\{ \begin{array}{c} 1.18 \times 10^{-6} \alpha^4 - 1.32 \times 10^{-4} \alpha^3 + 4.82 \times 10^{-1} \alpha^2 \\ -4.55 \times 10^{-2} \alpha + 0.879 \end{array} \right\} \\
\[ n = \left\{ \begin{array}{c} 5 \times 10^{-7} \alpha^4 - 5 \times 10^{-5} \alpha^3 + 1.98 \times 10^{-3} \alpha^2 \\ -2.06 \times 10^{-2} \alpha + 1.63 \end{array} \right\} \\
\]

Where \( Q = \frac{q}{2} \sqrt{\frac{2g}{3}} Lgh_x \), \( H = \frac{h}{h_x} \)

The same is obtained as an Excel curve fit as shown in Fig. 4.

It can be seen that the equation developed by the model agrees with the one obtained by Excel and further, the regression coefficient in both the cases is exactly unity. This improves the credibility of the analysis and practical usage of the notch. Even though the obtained discharge-head-inclination equation is complicated, it reduces to simple equation once the \( \theta \) values are substituted and simplified.

6. DISCHARGING CAPACITY

The discharging capacity of the inclined trapezoidal notch relative to normal position is shown in Fig. 4. If the discharge coefficient of normal weir is considered to be unity then the increase in discharging capacity of the same trapezoidal notch for various inclinations are as plotted in Fig. 4.

\[ C_m = 2E-06 \alpha^4 - 0.0001 \alpha^3 + 0.005 \alpha^2 - 0.0385 \alpha + 1 \quad (R^2 = 1) \]

\[ C_m = 0.8334e^{0.0256n} \quad (R^2 = 0.9126) \]

From Fig. 4, it is seen that the discharging capacity increases by over 384% relative to normal position of the notch. Further the increase can be seen to be exponential with regression coefficient being nearly unity (0.91) whereas for all the equation fit is unity. This validates the theory and experiments.

7. CONCLUSIONS:

Following conclusions were drawn based on the experimental investigation and the subsequent analysis by the authors.

- The discharging capacity of the weir increases with the increase in inclination of the plane of weir. In particular it is found to be maximum at 60\(^\circ\) inclination at 384.5\% of that of the normal position.
- The conventional weirs can be conveniently fixed inclined at any suitable inclination to the plane normal (to the flow axis) without much difficulty.
- The property of increase in discharge coefficient with increase in inclination of weir plane can be used to discharge more water quickly without increasing afflux on upstream side in pre-designed canal structure during flood season, without changing the pre-installed weir, which is practically very difficult.
- Larger area of flow is possible in the inclined notches relative to the conventional normally positioned notches. This reduces afflux on upstream of notch.
- Due to the simple geometry and ease of construction, inclined trapezoidal notch-weirs find its applications as a simple measuring devices in irrigation, chemical and sanitary engineering for flow measurement and flow control.
- The Mathematical modeling results in a single head-discharge-inclination equation which can be used for any trapezoidal notch of any desired inclination.
- Relatively increase in discharging capacity for the proposed inclined trapezoidal notch is greater than rectangular and lesser than trapezoidal notches. However, the limitation of fixing the weir perfectly with weir crest horizontal reduces the errors in discharge computations.

8. LIMITATION:

The experiment can be done with larger discharge in larger channels and the Head-Discharge-Inclination equation can be improved by using the model.

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REFERENCE:


