1. INTRODUCTION

During recent years, the flow of viscous fluids through a porous medium has been a subject of intensive studies because of its natural occurrence and importance in many industrial and engineering problems. The production of petroleum and natural gases, well drilling and lodging require many predictions based on results of fluid flow through a porous medium. The flow of blood through lungs and arteries are also examples of flow through porous media. The motion of the fluid is affected by so many factors. The boundaries of the fluid affect the flow to have stationary boundaries, fluctuating boundaries, moving boundaries, oscillatory boundaries and so on. The fluid motion in ducts, parallel plate channels, rectangular channels, parabolic boundaries, circular boundaries have been studied due to their importance in engineering and technology. At present, considerable attention has been given to the study of hydro magnetic convective flow of viscous fluids in rotating systems in connection with theories of fluid motion, two-phase flows, stratified flows, flow of immiscible fluids, flow through porous media, flow with suction/injection, flow in presence of heat source/heat flux, flow past a porous/hot/accelerated vertical plate, flow through channels of different shape with varied restrictions and so on. New ideas have been added to the literature to possible applications in geophysics, engineering problems, geothermal energy, stem stimulation of oil field, food drying and heat pipes.

The study of fluctuating flow was initiated by Stokes (1851). Lord Rayleigh (1911) obtained the flow due to an oscillatory plane wall in the same fluids. Lighthill (1954) analyzed the flow under fluctuating boundaries and developed mathematical models for fluid flows. Yih (1959) studied the effect of density variations on the fluid and has analyzed forced oscillations in a viscous stratified fluid in which the density and viscosity vary exponentially with vertical coordinates. Ahmadi and Manvi (1971) have given general equation of flow of fluid through porous media. Saffman (1962) studied stability of laminar flow of a dusty gas neglecting the volume friction of dust particles. Nayfeh (1966) has formulated the equation of motion of the fluid particles taking the volume fraction of the dust particles into account. Kishor et. al. (1981) studied hydrodynamic flow past an axial porous plate in rotating system. Singh & Tripathi (1988) have discussed problems on flow of viscous fluids through a porous medium in a rotating system. Free convective hydro magnetic flows in rotating system have been discussed by Prasadaraao et.al. (1981) and Senger et. al. (1987) past a porous plate. Singh & Kulshrestha (1993) has discussed MHD flow of viscous incompressible fluid between two co-axial rotating porous cylinders bounded by permeable beds. Singh et. al. (1999) have studied the effects of Hall current in free convective unsteady hydro magnetic boundary layer flow in rotating viscous fluid. Kumar (1994) have studied viscoelastic fluid flow rotating in a porous medium in presence of magnetic field. Singh & Kumar (1995) have discussed free convection in oscillatory MHD flow of the above problem for viscoelastic rotating liquid in a porous medium with constant heat source.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider the unsteady free convection flow of an incompressible, homogeneous, electrically conducting, and viscous liquid through a porous medium past an infinite vertical porous plate. A Cartesian co-ordinate system (x, y, z) rotating uniformly with the liquid in a rigid state of rotation with a constant angular velocity \((\Omega)\) about z-axis is considered. The vertical plate is assumed in the plane \(z=0\) and z-axis is taken normal to the plate pointing towards the medium. The flow is considered in the presence of a uniform magnetic field of intensity \(B_0\) applied normal to the flow and a time independent suction velocity \(w=\dot{w}_0(1+re^{int})\) has been taken into account at the plate (\(\dot{w}_0\) is a positive constant and negative sign indicates that the suction velocity towards the plate). In addition to the above, our analysis is based on the following assumptions:
1. The magnetic Reynolds number is very small so that induced magnetic field is ignored in comparison to the imposed magnetic field.
2. No external electric field is applied in the flow region so that the effect of polarization of ionized fluid is negligible.
3. The Hall Effect and viscous dissipation effect have been ignored.
4. Only electromagnetic body force (Lorentz force) is considered.

Therefore for the present configuration and under usual Boussinesq’s approximation, the governing equations are as follows:

Momentum equations:
\[
\frac{\partial u}{\partial x} - 2\Omega v + w_0(1+\epsilon^m)\frac{\partial u}{\partial z} = (T - T_w)g\beta + \frac{\partial^2 u}{\partial z^2} - \frac{u - \sigma B_y u}{\rho} \tag{1}
\]

\[
\frac{\partial v}{\partial x} + 2\Omega u + w_0(1+\epsilon^m)\frac{\partial v}{\partial z} = \frac{\partial^2 v}{\partial z^2} - \frac{v - \sigma B_y v}{\rho} \tag{2}
\]

Energy equation:
\[
\frac{\partial T}{\partial x} - w_0(1+\epsilon^m)\frac{\partial T}{\partial z} = \frac{\partial^2 T}{\partial z^2} - \frac{\partial^2 T}{\partial x^2} \tag{3}
\]

The boundary conditions are:
\[
t > 0: \quad u = v = 0, \quad \frac{\partial T}{\partial z} = \frac{-q}{K}, \quad \text{at} \quad z = 0
\]

\[
u, \nu \rightarrow 0, \quad T \rightarrow T_w, \quad \text{as} \quad z \rightarrow \infty \tag{4}
\]

Where \( u \) and \( v \) are velocities in \( x \) and \( y \) directions respectively, \( T \) is the temperature of the fluid, \( T_w \) is the temperature of the fluid far away from the plate, \( \beta^* \) is the volumetric coefficient of thermal expansion, \( K \) is the thermal conductivity, \( C_p \) is the specific heat at constant pressure, \( k \) is the porosity of the medium, \( q \) is the constant heat flux and the other symbols have their usual meanings. The following non-dimensional variables have been used:

\[
\begin{align*}
\tilde{u} & = \frac{u}{W}, & \tilde{v} & = \frac{v}{W}, & \tilde{T} & = \frac{T}{T_w}, & \tilde{\Omega} & = \frac{\Omega}{W}, & \tilde{\epsilon} & = \frac{\epsilon}{W}, & \tilde{\nu} & = \frac{\nu}{W}, & \tilde{K} & = \frac{K}{K_w}, & \tilde{\beta} & = \frac{\beta}{\beta^*},
\end{align*}
\]

Using the above variables the equations (1) to (3) (neglecting the stars over them) are reduced to:

\[
\frac{\partial \tilde{u}}{\partial \tilde{x}} - 2\tilde{\Omega} \tilde{v} + \tilde{w}_0(1+\tilde{\epsilon}^m)\frac{\partial \tilde{u}}{\partial \tilde{z}} = (\tilde{T} - \tilde{T}_w)g\tilde{\beta} + \frac{\partial^2 \tilde{u}}{\partial \tilde{z}^2} - \frac{\tilde{u} - \sigma \tilde{B}_y \tilde{u}}{\tilde{\rho}} \tag{5}
\]

\[
\frac{\partial \tilde{v}}{\partial \tilde{x}} + 2\tilde{\Omega} \tilde{u} + \tilde{w}_0(1+\tilde{\epsilon}^m)\frac{\partial \tilde{v}}{\partial \tilde{z}} = \frac{\partial^2 \tilde{v}}{\partial \tilde{z}^2} - \frac{\tilde{v} - \sigma \tilde{B}_y \tilde{v}}{\tilde{\rho}} \tag{6}
\]

and
\[
\gamma \frac{\partial \tilde{T}}{\partial \tilde{x}} + \tilde{w}_0(1+\tilde{\epsilon}^m)\frac{\partial \tilde{T}}{\partial \tilde{z}} = \frac{\partial^2 \tilde{T}}{\partial \tilde{z}^2} \tag{7}
\]

Assuming \( W = u + iv \), the equations (5) and (6) give

\[
\frac{\partial W}{\partial \tilde{x}} + 2i\tilde{\Omega} \tilde{W} + (1 + \tilde{\epsilon}^m)\frac{\partial W}{\partial \tilde{z}} = G \tilde{T} + \frac{\partial^2 \tilde{W}}{\partial \tilde{z}^2} - (M^2 + \frac{1}{k})\tilde{W} \tag{8}
\]

The boundary conditions (5) become:
\[
t > 0: \quad \tilde{W} = 0, \quad \frac{\partial \tilde{T}}{\partial \tilde{z}} = \frac{-q}{K}, \quad \text{at} \quad \tilde{z} = 0 \tag{9}
\]

\[
\tilde{W} = 0, \quad T \rightarrow T_w, \quad \text{as} \quad \tilde{z} \rightarrow \infty \tag{10}
\]

3. SOLUTION OF THE PROBLEM

On solving the equations (2.7) and (2.8), we obtain primary and secondary velocity as

\[
u = A^*_{\text{primary}} \quad \text{and} \quad \nu = A^*_{\text{secondary}} \tag{11}
\]

The rate of heat transfer is given by

\[
u = A^*_{\text{primary}} \quad \text{and} \quad \nu = A^*_{\text{secondary}} \tag{12}
\]

4. DISCUSSION OF THE RESULTS

Table-1 shows skin-friction \( (s\tau_w) \) due to primary velocity for cooling case \( (G_r > 0) \) at \( \epsilon = 0.002 \) to observe the effects of Prandtl number \( (P_r) \), magnetic parameter \( (M) \), permeability parameter \( (k) \), rotation parameter \( (E) \), Grashof number \( (G_r) \), frequency parameter \( (n) \) and time parameter \( (t) \). It is observed that an increase in \( k \), \( G_r \) or \( n \) leads to an increase in the skin-friction \( (s\tau_w) \) while an increase in \( P_r \), \( M \) or \( t \) decreases the skin-friction \( (s\tau_w) \).

Table-2 represents the skin-friction \( (s\tau_w) \) due to secondary velocity for cooling case \( (G_r > 0) \) at \( \epsilon = 0.002 \) to observe the effects of Prandtl number \( (P_r) \), magnetic parameter \( (M) \), permeability parameter \( (k) \), rotation parameter \( (E) \), Grashof number \( (G_r) \), frequency parameter \( (n) \) and time parameter \( (t) \). It is observed that an increase in \( k \), \( E \), \( G_r \) or \( t \) leads to an increase in the skin-friction \( (s\tau_w) \) while an increase in \( P_r \) or \( M \) increases the skin-friction \( (s\tau_w) \).

Table-3 shows skin-friction \( (s\tau_w) \) due to primary velocity for heating case \( (G_r < 0) \) at \( \epsilon = 0.002 \) to observe the
effects of Prandtl number ($P_r$), magnetic parameter (M), permeability parameter (k), rotation parameter (E), Grashof number ($G_r$), frequency parameter (n) and time parameter (t). It is observed that an increase in k, $G_r$ or n leads to a decrease in the skin-friction ($\tau_r$) while an increase in $P_r$, M, E or t increases the skin-friction ($\tau_r$). Table-4 represents the skin-friction ($\tau_r$) due to secondary velocity for heating case ($G_r < 0$) at $\varepsilon = 0.002$

to observe the effects of Prandtl number ($P_r$), magnetic parameter (M), permeability parameter (k), rotation parameter (E), Grashof number ($G_r$), frequency parameter (n) and time parameter (t). It is observed that an increase in k, $G_r$, n or t leads to an increase in the skin-friction ($\tau_r$) while an increase in $P_r$ or M decreases the skin-friction ($\tau_r$).

### Table-1: Skin-friction Due to Primary Velocity
(Cooling Case at $E=0.002$)

<table>
<thead>
<tr>
<th>$P_r$</th>
<th>M</th>
<th>k</th>
<th>E</th>
<th>$G_r$</th>
<th>n</th>
<th>t</th>
<th>$\tau_r$</th>
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### Table-2: Skin-friction Due to Secondary Velocity
(Cooling Case at $E=0.002$)

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### Table-3: Skin-friction Due to Primary Velocity
(Heating Case at $E=0.002$)

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### Table-4: Skin-friction Due to Secondary Velocity
(Heating Case at $E=0.002$)

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<th>$P_r$</th>
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### NOMENCLATURE

- $x$, $y$, $z$: Cartesian coordinates
- $u$, $v$: components of velocity
- $t$: time
- $w_s$: suction velocity
- $U_0$: free stream velocity
- $L_v$: dimensionless constant velocity of the plate
- $k_0$: constant permeability of the medium
- $k(t)$: Dimensionless time dependent permeability of the medium
- $H_0$: constant magnetic field
- $g$: acceleration due to gravity
- $K$: thermal conductivity
- $C_p$: specific heat at constant pressure
- $T$: temperature
- $T_w$: wall temperature
- $T_a$: temperature away from the plate
- $Q$: constant heat source
- $N$: frequency of oscillation
- $\Omega$: uniform angular velocity about $z$-axis

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GREEK SYMBOLS

\( \sigma \) = Electrical conductivity of the liquid
\( \mu \) = Kinematic coefficient of viscosity
\( \beta \) = Volumetric coefficient of thermal expansion
\( \mu_e \) = Magnetic permeability
\( \rho \) = Density of liquid
\( \tau_w \) = Skin friction from surface of the plate
\( N_u \) = Nusselt number
\( P_r \) = Prandtl number

REFERENCES