

## PERFORMANCE PREDICTION OF STRAIGHT BLADED DARRIEUS WIND TURBINE BY SINGLE STREAMTUBE MODEL

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### ABSTRACT

There are two categories of modern wind turbines, namely horizontal axis wind turbines (HAWTs) and vertical axis wind turbines (VAWTs), which are used mainly for electricity generation and pumping water. The main advantage of VAWT is its single moving part (the rotor) where no yaw mechanisms are required, thus simplifying the design configurations significantly. Scientists have developed numerous designs based on several aerodynamic computational models. These models are crucial for deducing optimum design parameters and also for predicting the performance of VAWT. In this review, the single streamtube aerodynamic model that has been used for performance prediction and design of straight-bladed Darrieus-type VAWT is discussed. Single stream tube model can predict the coefficient of performance easily before experiment of the turbine. In this paper co-efficient of torque by single stream tube model is discussed.

**KEYWORDS:** Single stream tube model, Darrieus turbine, Aerodynamic model, VAWT

### 1. INTRODUCTION

The Darrieus vertical axis wind turbine concept attracted considerable research interest in the years of 1970s and 1980s, but has never competed successfully with horizontal axis wind turbines. In recent years there has been growing interest in Darrieus straight blade wind turbine for low head application which is based on Darrieus wind turbine.



**Figure 1: Darrieus Wind Turbine**

In 1974, Templin proposed the single stream tube model which is the first and most simple prediction method for the calculation of Aerodynamic performance characteristics of curve blade Darrieus Vertical axis wind turbine [1]. Stream tube models are momentum models based on Glauert's Blade element theory[2]. In stream tube models the change in fluid momentum in the flow direction is equated to the stream wise forces on the aerofoil blades.

In this model the entire turbine is assumed to be enclosed within a single stream tube. The objective of the model is to determine the performance coefficient of rotor.

### 2. NOMENCLATURE

- $A$  Projected frontal area of turbine  
 $C$  Blade chord  
 $C_d$  Blade drag coefficient  
 $C_{dor}$  Reference zero-lift-drag coefficient  
 $C_D$  Turbine overall drag coefficient  
 $= F_D / \rho AV_\infty^2$   
 $C_{DD}$  Rotor drag coefficient  $= F_D / \rho AV_a^2$   
 $C_l$  Blade lift coefficient  
 $C_n$  Normal force coefficient  
 $C_p$  Turbine overall power co-efficient=  
 $P / \rho AV_\infty^3$

$C_T$  Turbine overall torque coefficient=  
 $T_B / \rho AV_\infty^2 R$

$C_t$  Tangential force coefficient

$D$  Blade drag force

$F_n$  Normal force

$F_t$  Tangential force

$H$  Height of turbine blade

$L$  blade lift force

$N$  Number of blade

$R$  Turbine radius

$Re$  Local Reynolds number  $= \rho WC / \mu$

$V_a$  Induced velocity

$V_c$  or  $V_t$  Chordal velocity component

$V_n$  Normal velocity component

$V_w$  Wake velocity

$W$  Relative velocity

$\rho$  Density of fluid

$\theta$  Azimuth position of blade

$\alpha$  Angle of attack

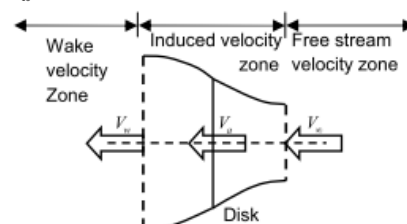
### 3. AXIAL INDUCTION FACTOR

The axial induction factor, 'a' is the fractional decrease in wind velocity between the free stream and the rotor plane, so it is defined as [11],

$$a = \frac{Nc R\omega}{2\pi R V_\infty} \sin \theta \quad (1)$$

We can define induction factor by reference [5],

$$a = \frac{V_\infty - V_a}{V_\infty} \quad (2)$$



**Figure 2: Actuator disc model for Darrieus rotor.**

The value for  $V_a$  can be obtained by Gluert Actuator Disk theory [2], the expression of the uniform velocity through the rotor is,

$$V_a = \frac{V_\infty + V_w}{2} \quad (3)$$

From equation (1) and (2) we can find induced and wake velocity,

$$V_a = V_\infty(1-a) \quad (4)$$

$$V_w = V_\infty(1-2a)$$

(5)

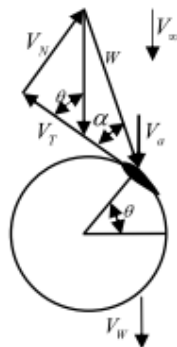
### 3. BLADE ANGLE OF ATTACK AND RELATIVE VELOCITY

The flow velocities in the upstream and downstream sides of the Darrieus rotor are constant as seen in figure 3. From this figure one can observe that the flow is considered to occur in the axial direction. The tangential velocity (or chordal velocity) component  $V_t$  (or  $V_c$ ) in tangential direction of blade profile and the normal velocity component  $V_n$  is normal to blade profile.

Tangential velocity ( $V_t$ ) and Normal velocity ( $V_n$ ) of blade which is given by [3],

$$V_t = R\omega + V_a \cos \theta \quad (6)$$

$$V_n = V_a \sin \theta \quad (7)$$



**Figure: 3: Flow velocities of Darrieus wind turbine.**

Angle of attack  $\alpha$  is angle between relative velocity  $W$  and tangential velocity  $V_t$  is obtained from the following expressions:

$$\alpha = \tan^{-1} \left( \frac{V_n}{V_t} \right) \quad (8)$$

Substituting the values of (6) and (7) in equation (8),

And Non-dimensional zing the equation

$$\alpha = \tan^{-1} \left[ \frac{\sin \theta}{(R\omega / V_a) + \cos \theta} \right] \quad (9)$$

The relative flow velocity ( $W$ ) can be obtained as,

$$W = \sqrt{V_t^2 + V_n^2} \quad (10)$$

Inserting the values of  $V_t$  and  $V_n$ , and non-dimensional zing, one can find velocity ratio as,

$$\frac{W}{V_a} = \sqrt{\left( \frac{R\omega}{V_a} \right)^2 + 2 \left( \frac{R\omega}{V_a} \right) \cos \theta + 1} \quad (11)$$

Local relative wind dynamic pressure ( $q$ ) is given by:

$$q = \frac{1}{2} \rho W^2 \quad (12)$$

### 4. BLADE ELEMENT FORCE AND DRAG CO-EFFICIENT

Since the disk acts as a drag device, the source of drag must be a pressure difference across the disk and this drag manifests itself as thrust loading along the axis normal to the disk. Rewriting N2 in terms of momentum:

$$\sum F_x = m(\Delta V) \quad (13)$$

According to glauert's theory [2] the velocity through a wind mill disk  $V_a$  is the arithmetic mean of the undisturbed velocity  $V_\infty$  and the velocity in the wake. The turbine drag  $D$  is given by:

$$D = m(V_\infty - V_a) \quad (14)$$

$$D = 2\rho AV_a(V_\infty - V_a) \quad (15)$$

The disk drag coefficient  $C_{DD}$  [4] based on dynamic pressure and the disk area is defined as:

$$C_{DD} = \frac{D}{\frac{1}{2} \rho AV_a^2} \quad (16)$$

And from equation (13) and (14)

$$C_{DD} = 4 \left( \frac{V_\infty}{V_a} - 1 \right) \quad (17)$$

$$\frac{V_\infty}{V_a} = 1 + \frac{1}{4} C_{DD} \quad (18)$$

For structural design purpose, a more convenient drag co-efficient  $C_D$  is based on the ambient dynamic pressure [4]:

$$C_D = \frac{D}{\frac{1}{2} \rho V_\infty^2 A} \quad (19)$$

$$C_D = C_{DD} \left( \frac{V_D}{V_\infty} \right)^2 \quad (20)$$

$$C_D = \frac{C_{DD}}{\left( 1 + \frac{1}{4} C_{DD} \right)^2} \quad (21)$$

For a given turbine geometry and rotational speed, turbine power and rotor drag are calculate using the blade element theory [4]. To calculate the blade element forces, the local Winddynamic angle of attack and the local relative dynamic pressure are required. The resultant velocity vector, relative vector, relative to the moving blade element, is resolved into two perpendicular components; one parallel to the span wise and other in blade profile plane as shown in figure 4. The velocity component parallel to the blade span wise has no effect on the blade element Winddynamic forces.

Assuming that the instantaneous local blade element lift and drag coefficients  $C_l$  and  $C_d$  are function of angle of attack in steady flow, the normal force coefficient  $C_n$  and tangential force coefficient  $C_t$  are given by following expressions:

$$C_t = C_l \sin \alpha - C_d \cos \alpha \quad (22)$$

$$C_n = C_l \cos \alpha + C_d \sin \alpha \quad (23)$$

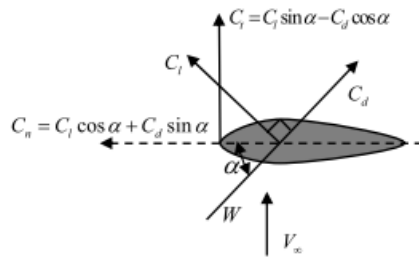


Figure 4: Normal and Tangential force coefficient on blade

The net tangential and normal forces defined as:

$$F_t = C_t \frac{1}{2} \rho C H W^2 \quad (24)$$

$$F_n = C_n \frac{1}{2} \rho C H W^2 \quad (25)$$

Blade element of chord  $C$  is subjected to an elemental normal force  $dN$  and a forward thrust force  $dT$  given by the relation:

$$dF_n = C_n q C H \quad (26)$$

$$dF_t = C_t q C H \quad (27)$$

The elemental drag at any blade position ' $\theta$ ' as shown in figure 5 is:

$$dD = (dF_n \sin \theta - dF_t \cos \theta) d\theta \quad (28)$$

The total drag value is obtained by integration on a full revolution ( $0 \leq \theta \leq 2\pi$ ). Thus the total drag of a rotor with  $N$  blades of constant chord  $C$  is given by the relation:

$$D = \frac{NCH}{2\pi} \int_0^{2\pi} q (C_n \sin \theta - C_t \cos \theta) d\theta \quad (29)$$

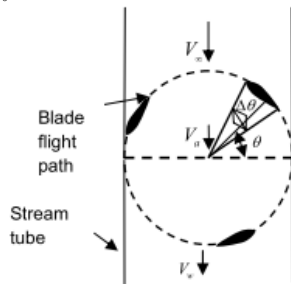


Figure 5: Blade at  $\theta$  azimuth position.

Drag co-efficient is given by,

$$C_D = \frac{D}{\frac{1}{2} \rho V_\infty^2 A} \quad (30)$$

Put the value of drag  $D$  from equation (29).

$$C_{DD} = \frac{NC}{\pi D \rho V_\infty^2} \int_0^{2\pi} q (C_n \sin \theta - C_t \cos \theta) d\theta \quad (31)$$

For numerical integration of the expression of above equation is divided into a finite number of surface element having equal increments of azimuth angle  $\theta$ .

### 5. ROTOR TORQUE AND POWER CO-EFFICIENT

The rotor torque is produced by only the tangential component of the force on the rotor blade element

and in its elemental form, for a single blade element height ' $H$ ' and at  $d\theta$  azimuth position of the blade the rotor torque is given by:

$$dT_s = CHRqC_t d\theta \quad (32)$$

Blade torque varies with blade azimuthally angle  $\theta$ . The total torque for rotor is obtained by integration on a full revolution ( $0 \leq \theta \leq 2\pi$ ).

$$T_B = \frac{NCHR}{2\pi} \int_{\theta=0}^{2\pi} q C_t d\theta \quad (33)$$

Torque co-efficient is defined as:

$$C_T = \frac{T_B}{\frac{1}{2} \rho V_\infty^2 AR} \quad (34)$$

From equation (34) and (33),

$$C_T = \frac{NC}{2\pi \rho R V_\infty^2} \int_0^{2\pi} q C_t d\theta \quad (35)$$

The shaft power is then,

$$P = \omega T_B \quad (36)$$

For straight blade vertical axis rotor of diameter  $D$  and height  $H$  only one part of the total wind kinetic energy flow is converted into useful shaft power. Maximum possible power of the swept rotor area is:

$$P_{max} = \frac{1}{2} \rho V_\infty^3 A \quad (37)$$

The power coefficient  $C_p$  can be defined as the ratio of actual power  $P$  given by equation to the maximum value  $P_{max}$  given by [5],

$$C_p = \frac{P}{P_{max}} \quad (38)$$

Put value of  $P_{max}$  and  $P$  in above equation,

$$C_p = \left( \frac{NC\omega}{2\pi \rho V_\infty^3} \right) \int_0^{2\pi} q C_t d\theta \quad (39)$$

Coefficient of power for rotor is obtained by integration on a full revolution ( $0 \leq \theta \leq 2\pi$ ). The relation with coefficient of torque and coefficient of power are given by:

$$C_p = \lambda C_T \quad (40)$$

We can find co-efficient of power from above equation for a given turbine geometry and for each specified value of tip speed ratio  $R\omega / V_a$

### 6. CONCLUSION AND FUTURE WORK

Single stream tube model is simple prediction of coefficient of performance of rotor. Time require for calculation is less. This model can predict the overall performance of low tip speed ration of rotor and a lightly loaded turbine but according to the inquest, it always predicts higher power than the experimental results. It does not predict the wind velocity variations across the rotor. These variations gradually increase with the increase of the blade solidity and tip speed ratio. Single stream tube model does not take into account the difference in the induced velocities between the upstream and downstream halves of the rotor or any difference in velocities across the rotor such as

those due to wind shear. The main drawback of these models is that they become invalid for large tip speed ratios and also for high rotor solidities because the momentum equations in these particular cases are inadequate.

In future work, another model will be developed which can accurately predict the performance coefficient for darrieus rotor for higher tip speed ratio and higher solidity rotor.

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