OPTIMUM POWER ALLOCATION AND SYMBOL ERROR RATE (SER) PERFORMANCE OF VARIOUS SPACE TIME BLOCK CODES (STBC) OVER FAADING COGNITIVE MIMO CHANNELS IN DIFFERENT WIRELESS ENVIRONMENT

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ABSTRACT
A cognitive radio (CR) is a transceiver which automatically detects the available unused channels in the wireless spectrum. A cognitive radio network (CRN) is formed by either allowing the secondary users (SUs) to coexist with the primary users (PUs). In this paper, we focus our work on the power allocation for fading MIMO channels with statistical Channel State Information (CSI). Since the ergodic capacity function is very complex, the ergodic capacity maximization is rather hard (L. Zhang 2008, Caire.G, 1999).

The rest of this paper is organized as follows: Section II provides the system model of CR MIMO networks. Section III gives the proposed algorithms. The simulation results and conclusions are given in Sections IV and V respectively.

The following notations are used in this paper. |.| denotes the determinant; (.)\(^T\) denotes the transpose of the matrix; E(.) denotes the statistical expectation; Tr(.) denotes the trace of the matrix. The identity matrix is denoted by I.

II. SYSTEM MODEL
This paper considers the CR networks with one secondary transmitter-receiver pair shares the spectrum with the primary radio networks, which consists of N number of Primary Transmitters (PT) and K number of Primary Receivers (PR). Fig. 1 shows the block diagram of the proposed method. This block diagram explains the flow of STBC and modulation schemes for MIMO channels. We assume that there are Np number of receive antennas at each PR, and Mr number of transmit antennas at each PT, Nr receive antennas at each Secondary Receiver (SR), and Mt transmit antennas at each Secondary Transmitter (ST). Since the primary users and the secondary users simultaneously transmit in the same bandwidth, the received signal at SR can be expressed as

\[ Y = HX + \sum_{i} T_i X_p,i + N_o \]

Where H denotes the channel matrix from ST to SR, Ti denotes the channel matrix from the \(i^{th}\) PT to SR, Xi is the transmitted signal vector at ST, Xp,i is the transmitted signal vector at the \(i^{th}\) PT, and No is the normalized additive white complex Gaussian noise vector with zero mean and variance of \(\sigma_n^2\). The capacity of the SU link is given as

\[ C_s = \log_2 |I + R^*HQH^*| \]

Where Q is the transmit covariance matrix of ST, R is the noise plus interference covariance matrix at ST.

Then the SU capacity maximization problem is given by

\[ \text{maximize } \log_2 |I + R^*HQH^*| \]
subject to $\text{Tr}(Q) < P_T$

Where $P_T$ is the maximum total transmit power of ST.

**III. POWER ALLOCATION ALGORITHMS**

Since $Q$ is a positive semi definite matrix, we can express $Q$ into its Eigen value Decomposition (ED) as $Q=\mathbf{F} \mathbf{A}^{1/2} \mathbf{A}^{1/2} \mathbf{F}^*$, where $\mathbf{A} \triangleq \text{diag}(P_1, P_2, \ldots, P_M)$ where $P_1, P_2, \ldots, P_M$ are the individual transmitted antenna power. In MIMO signal processing (Junling Mao et al 2012), we also call $\mathbf{F}$ as the precoding matrix.

Substituting the ED of $Q$ into the objective function we have

$$E (\log_2 |I+R^1HQH^T|) = E (\log_2 (I+R^1HF)A(HF)^*)$$

(3)

**A. Lagrangian Multiplier Algorithm**

One upper bound of the ergodic capacity can be given by

$$E (\log_2 |I+R^1HQH^T|) \leq E (\log_2 |I+E(R^1)HAH^T|)$$

(4)

Where $E(R^1)$ is a diagonal matrix.

One lower bound of the ergodic capacity can be given by

$$E (\log_2 |I+R^1HQH^T|) \geq E (\log_2 |I+E(R^1)^*HAH^T|)$$

(5)

Where $E(R^1)^*$ is a diagonal matrix and its diagonal elements have the same value. The elements of $\mathbf{H}$ are distributed as statistically independent identically distributed (i.i.d.) and $\mathbf{F}$ is a unitary matrix, so the elements of $\mathbf{HF}$ are also distributed as i.i.d.

According to the aforementioned analysis, we give the following iteration procedure for Lagrangian Multiplier algorithm to solve the power problem. The loop is used to solve the Lagangian dual problem. $t_1$ and $t_2$ are the step length of the loop. Lagrangian Multiplier Algorithm also converges to the optimal point within a small range. If the gradient of the ergodic capacity function is known (or obtained by Monte Carlo simulation) the framework of Lagrangian Multiplier Algorithm can be used to solve the optimal power allocation.

Table 1 gives the comparison of closed formula and Monte Carlo constants for various antenna configurations.

<table>
<thead>
<tr>
<th>$\Lambda(P_i)$</th>
<th>$\Psi_{(q_0, \Lambda)}$ (Monte Carlo)</th>
<th>$\Psi_{(q_0, \Lambda)}$ (Closed formula)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.7210</td>
<td>0.7219</td>
</tr>
<tr>
<td>3.0</td>
<td>0.3619</td>
<td>0.3613</td>
</tr>
<tr>
<td>7.0</td>
<td>0.1804</td>
<td>0.1804</td>
</tr>
<tr>
<td>10.0</td>
<td>0.1312</td>
<td>0.1312</td>
</tr>
</tbody>
</table>

Algorithm:

Initialization: $U > 0, V_k > 0, P_i > 0$

Repeat

1. Update $F$

2. Repeat

$$P_i = (P_i + t_1 \times (\Psi_{(q_0, \Lambda)} - \sum_i V_k ||G_k f_i||))$$

Until all $P_i$ converge

3. Update $U$

$$U = (U + t_2 \times (P_T - \sum_i P_i))$$

$$V_k = (V_k + t_2 \times (||G_k f_i|| ||P_i||))$$

Until $U, V_k$ converge

With the value of $\Psi_{(q_0, \Lambda)}$ and $F$, the gradient of the maximization objective function can be solved, and optimal $P_i$ can also be calculated.

**B. Water Filling Algorithm**

Reset Sub channels $S_1, S_2, \ldots, S_k$

Initialize $K$

Compute WF Energies

Compute WF constant

Select Lowest energy

Reset Sub channels $S_1, S_2, \ldots, S_k$

Fig. 2 Flow diagram of Water filling algorithm.
Although Lagrangian Multiplier Algorithm can find a nearly optimal solution it is very complex. In this section we intend to propose a low complexity algorithm to solve another bound maximization problem with (Gastpar M 2007). Fig. 2 shows the flow diagram of water filling algorithm which is based on channel power or energy. Unlike the upper bound given by (4), this upper bound is derived from both the statistical expectation of H and R⁻¹. It can make the optimization problem easier.

We get a bound maximization problem as

\[
\max \sum_i \|G_i f_i\| P_i \leq P_t
\]

\(P_i > 0\) for all i

**Water filling Algorithm Steps:**
We do not need to reorder the MIMO-OFDM sub channel gain realization in a descending order.
- Take the inverse of the channel gains.
- Water filling has non uniform step structure due to the inverse of the channel gain.
- Initially take the sum of the Total Power P\(_t\) and the Inverse of the channel gain. It gives the complete area in the water filling and inverse power gain.
- Decide the initial water level by the formula given below by taking the average power allocated (average water level)
- The power values of each sub channel are calculated by subtracting the inverse channel gain of each channel.
- In case the Power allocated value becomes negative stop the iteration process.

**Algorithm:**
**Initialization**
Number of Channels:
\(m = N_t = N_r = 2\) and \(N_s = 1, N_s = 7\);
Total transmitted power:
\(P_t = 10\) dB, 20dB, 30dB.
Equal power distribution
\(P_i = \frac{P_t}{m}; \quad i = 1, 2\)
Capacity \(C = B \times \log_2 (1 + P_i)\) bits/sec.
Waterfilling capacity
\(C_{wf} = \frac{1}{2} \sum_i \log_2 (1 + P_i/N_i)\) bits/sec.

Since the Lagrangian dual problem holds the strong duality, Water filling Algorithm can converge to the optimal point within a small range.

**IV. SIMULATION RESULTS**
In this section, we consider the cognitive radio network where a secondary transmitter–receiver pair coexists with two primary transmitter–receiver pairs. The antenna configuration of the secondary system is set as \(M_s = N_r = 2\). We change \(P_t/N_0\) at the secondary transmitter from 0 to 30 dB. Numerical results are given to compare the average capacity of Shannon’s capacity, MIMO with \(N_T = N_R = 2\), Lagrangian multiplier algorithm and Waterfilling algorithm. Here four different STBC codes are taken for wireless channel for simulation.

Symbol Error Rate (SER) for fading channels are calculated based on Space Time Block Codes and Modulation schemes such as QAM, BPSK, QPSK and MFSK and the simulated results are compared with the theoretical values. The average symbol energy \(E_{avg}\) for QAM modulation is given by,

\[
E_{avg} = \frac{1}{M} \sum_{i=1}^{M} (a_i^2 + b_i^2)
\]

With the definition of energy in mind, symbol error is approximated by,

\[
P_e \approx \frac{2}{\sqrt{M}} \left( 1 - \frac{1}{\sqrt{M}} \right) \text{erfc} \left( \sqrt{\frac{2E_{avg}}{\sqrt{2(M-1)N_o}}} \right)
\]

Where \(E_{avg}\) is calculated by equation (8).

The Symbol error rate for QPSK is given by,

\[
P_e = \text{erfc} \left( \frac{E}{\sqrt{2N_o}} \right)
\]

This brings up the distinction between symbol error and bit error.

The Symbol error rate for BPSK modulation is given by,

\[
P_{el} = P \left( \sqrt{E_{avg}} + n < 0 \right)
\]

Where \(n\) is Gaussian with mean 0 and variance \(N_o/2\).

The error expression for MFSK modulation with the usual notation is given by,

\[
P_e \leq \frac{1}{2} (M - 1) \text{erfc} \left( \sqrt{\frac{E}{2N_o}} \right)
\]

MFSK is different from MPSK in that each signal sits on an orthogonal axis (basis).

**Fig. 3** Symbol Error Rate for Rayleigh Fading Channel using QAM for different STBC Techniques
Fig. 3 shows the Symbol Error Rate (SER) for Rayleigh fading channel using QAM for different STBC techniques. From the analysis of the result, Golden code has minimum SER at SNR=20dB.

**Fig. 4** Symbol Error Rate for Nakagami Fading Channel using QAM for different STBC Techniques
Fig. 4 shows the Symbol Error Rate (SER) for Nakagami fading channel using QAM for different STBC techniques. From the analysis of the result, V-Blast code has minimum SER at SNR=20dB.

**Fig. 5** shows the Symbol Error Rate (SER) for Rician fading channel using QAM for different STBC techniques. From the analysis of the result, Silver code has minimum SER at SNR=20dB.
Fig. 5 shows the performance of Symbol Error Rate (SER) for Silver code under different modulations for Nakagami fading channel. From the analysis of the result, BPSK modulation has minimum SER upto SNR=16dB.

Fig. 6 shows the performance of Symbol Error Rate (SER) for Golden code under different modulations for Rayleigh fading channel. From the analysis of the result, QAM modulation has minimum SER upto SNR=30dB.

Fig. 7 shows the performance of Symbol Error Rate (SER) for V-Blast under different modulations for Nakagami fading channel. From the analysis of the result, QPSK modulation has minimum SER upto SNR=30dB.

Fig. 8 shows the performance of Symbol Error Rate (SER) for Silver code under different modulations for Nakagami fading channel. From the analysis of the result, BPSK modulation has minimum SER upto SNR=16dB.
comparing these algorithms water filling algorithm reaches maximum capacity. Table 2 compares the channel capacity of the system without using STBC and with using STBC codes. From that we clearly understand STBC with different modulation schemes gives very good performance in channel capacity for all the three wireless channels.

Table 2 Compares the Channel Capacity of previous and proposed algorithms.

<table>
<thead>
<tr>
<th>Channel Type</th>
<th>Channel Capacity with STBC and Modulation Schemes (bits/s/Hz)</th>
<th>Channel Capacity with STBC and Modulation Schemes (bits/s/Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rayleigh Fading Channel</td>
<td>25</td>
<td>35</td>
</tr>
<tr>
<td>Nakagami Fading Channel</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>Rician Fading Channel</td>
<td>20</td>
<td>35</td>
</tr>
</tbody>
</table>

V. CONCLUSION
Space time block code (STBC) is a technique used in wireless communication to transmit multiple copies of data stream across a number of antennas. In this paper, with the assumption of primary communication and secondary communication are going in a simultaneous manner. Based on the STBC code, the symbol error rate (SER) is calculated for various modulation schemes. According to the SER, Lagrangian multiplier algorithm and water filling algorithm are proposed to maximize the ergodic capacity.

From the performance analysis, we find that water filling algorithm can approach maximum capacity. Further it is recommended for practical systems because it can achieve the better tradeoff between complexity of algorithm and throughput performance.

REFERENCES