



Research Paper

OPTIMUM POWER ALLOCATION AND SYMBOL ERROR RATE (SER) PERFORMANCE OF VARIOUS SPACE TIME BLOCK CODES (STBC) OVER FADING COGNITIVE MIMO CHANNELS IN DIFFERENT WIRELESS ENVIRONMENT

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ABSTRACT –

A cognitive radio (CR) is a transceiver which automatically detects the available unused channels in the wireless spectrum. A cognitive radio network (CRN) is formed by either allowing the secondary users (SUs) to coexist with the primary users (PUs). In this paper, the ergodic capacity maximization problem is studied in the Rayleigh fading, Nakagami fading and Rician fading Multiple Input and Multiple Output (MIMO) channel and provides the optimum power allocation to achieve the ergodic capacity and outage capacity. This study includes the Symbol Error Rate (SER) performance of various Space Time Block Codes (STBC) such as Alamouti code, V-Blast code, Silver code, Golden code for different wireless channels such as Rayleigh, Nakagami and Rician with various modulation schemes. Numerical results are presented to evaluate the power allocation scheme for Lagrangian multiplier algorithm and Water filling (WF) algorithm schemes for comparison.

KEYWORDS Cognitive Radio (CR), ergodic capacity, Multiple Input Multiple Output (MIMO), Power allocation.

1. INTRODUCTION

The Cognitive Radio (CR) technique was first introduced by Mitola in his pioneering work (Mitola.J and Maguire .G.Q. 1999) and has drawn considerable attention due to its advantages of spectrum reusing. In CR networks, the secondary user (SU) usually communicates over the bandwidth originally allocated to the primary network. Spectrum handoff procedures occur when the primary users appear in the licensed band temporary occupied by the cognitive radio (CR) users and aim to help the CR users to vacate the spectrum rapidly and find available channel to resume the transmission. This process is also known as Dynamic spectrum management.

MIMO, or multiple input multiple output (Dongming Wang et al 2013), is a technique where multiple antennas are used at both the transmitter and the receiver to increase the link reliability, the spectral efficiency, or both.

The performance of MIMO channels can be improved with the help of Space Time Trellis code (Ilesanmi Banjo Oluwafemi 2013). According to the channel power, it is complex to compute the channel capacity. In this paper, Space Time Block Code (STBC) is analyzed for fading MIMO channels which makes simplicity to compute the channel capacity. The different digital modulation schemes such as BPSK, QPSK, QAM & MFSK are used to calculate the Symbol Error Rate (SER). By combining STBC and Modulation schemes, the capacity of MIMO channels are calculated.

In general a signal propagated between a transmitter and a receiver is often affected by fading (Y.-C. Liang et al 2006). One major source of fading is multipath propagation, where different copies of the signal partially cancel each other out at certain times and points in space. This increase in errors decreases the effective throughput of the signal (Musavian and S. Aissa 2007), thereby weakening the RF link. Here, we have considered the three wireless channels such as Rayleigh, Nakagami and Rician for secondary users' seamless communication (Yingbin Liang and Venugopal V. Veeravalli 2004, ZhangQ.T. 2003, Chengshan Xiao et al 2006).

In this paper, we focus our work on the power allocation for fading point-to-point cognitive MIMO channel with statistical Channel State Information

(CSI). Since the ergodic capacity function is very complex, the ergodic capacity maximization is rather hard (L. Zhang 2008, Caire.G, 1999).

The rest of this paper is organized as follows: Section II provides the system model of CR MIMO networks. Section III gives the proposed algorithms. The simulation results and conclusions are given in Sections IV and V respectively.

The following notations are used in this paper. $|\cdot|$ denotes the determinant; $(\cdot)^T$ denotes the transpose of the matrix; $(\cdot)^*$ denotes the complex conjugate of the matrix; $E(\cdot)$ denotes the statistical expectation; $\text{Tr}(\cdot)$ denotes the trace of the matrix. The identity matrix is denoted by \mathbf{I} .

II. SYSTEM MODEL

This paper considers the CR networks with one secondary transmitter-receiver pair shares the spectrum with the primary radio networks, which consists of N number of Primary Transmitters (PT) and K number of Primary Receivers (PR). Fig. 1 shows the block diagram of the proposed method. This block diagram explains the flow of STBC and modulation schemes for MIMO channels. We assume that there are N_p number of receive antennas at each PR, and M_p number of transmit antennas at each PT, N_s receive antennas at each Secondary Receiver (SR), and M_s transmit antennas at each Secondary Transmitter (ST). Since the primary users and the secondary users simultaneously transmit in the same bandwidth, the received signal at SR can be expressed as

$$Y_s = HX_s + \sum_i T_i X_{p,i} + N_o \quad (1)$$

Where H denotes the channel matrix from ST to SR, T_i denotes the channel matrix from the i^{th} PT to SR, X_s is the transmitted signal vector at ST, $X_{p,i}$ is the transmitted signal vector at the i^{th} PT, and N_o is the normalized additive white complex Gaussian noise vector with zero mean and variance of σ_H^2 .

The capacity of the SU link is given as

$$C_s = \log_2 |I + R^{-1}HQH^*| \quad (2)$$

Where Q is the transmit covariance matrix of ST, R is the noise plus interference covariance matrix at SR.

Then the SU capacity maximization problem is given by

$$\text{maximize } \log_2 |I + R^{-1}HQH^*|$$

subject to $Tr(Q) < P_T$

Where P_T is the maximum total transmit power of ST.

III. POWER ALLOCATION ALGORITHMS

Since Q is a positive semi definite matrix, we can express Q into its Eigen value Decomposition (ED) as $Q=FAF^*$, where $\Lambda=diag(P_1, P_2, \dots, P_{M_s})$ where P_1, P_2, \dots, P_{M_s} are the individual transmitted antenna

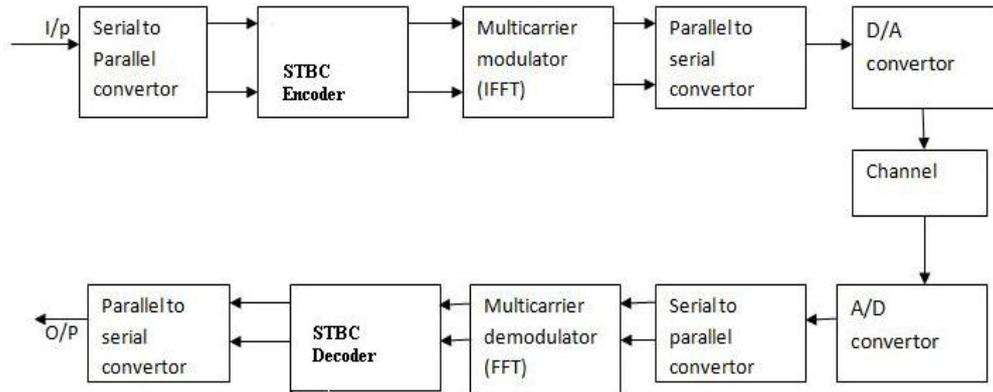


Fig. 1. Block diagram of MIMO system with STBC and modulation schemes.

Then it can be transformed into equivalent problem as

$$\text{maximize } E(\log_2|I+R^{-1}HQH^*|)$$

$$\text{Subject to } \sum_{i=1}^{M_s} P_i < P_T$$

$$P_i > 0 \text{ for all } i$$

Where P_i is the initial transmitted power. Secondary users capacity maximization is convex for P_i but nonconvex for F , so it is hard to directly solve.

In this section, we intend to maximize some bounds of the objective function. Although solving bound maximization problems (Junling Mao et al 2012) cannot get optimal power solution, it can get some nearly optimal power solutions that are close to the optimal power solution.

A. Lagrangian Multiplier Algorithm

One upper bound of the ergodic capacity can be given by

$$E(\log_2|I+R^{-1}HQH^*|) \leq E(\log_2|I+E(R^{-1})HAH^*|) \quad (4)$$

Where $E(R^{-1})$ is a diagonal matrix.

One lower bound of the ergodic capacity can be given by

$$E(\log_2|I+R^{-1}HQH^*|) \geq E(\log_2|I+E(R)^{-1}HAH^*|) \quad (5)$$

Where $E(R)^{-1}$ is a diagonal matrix and its diagonal elements have the same value. The elements of H are distributed as statistically independent identically distributed (i.i.d.) and F is a unitary matrix, so the elements of HF are also distributed as i.i.d.

According to the aforementioned analysis, we give the following iteration procedure for Lagrangian Multiplier algorithm to solve the power problem. The loop is used to solve the Lagrangian dual problem. t_1 and t_2 are the step length of the loop. Lagrangian Multiplier Algorithm also converges to the optimal point within a small range. If the gradient of the ergodic capacity function is known (or obtained by Monte Carlo simulation) the framework of Lagrangian Multiplier Algorithm can be used to solve the optimal power allocation.

Table 1 gives the comparison of closed formula and Monte Carlo constants for various antenna configurations.

power. In MIMO signal processing (Junling Mao et al 2012), we also call F as the precoding matrix.

Substituting the ED of Q into the objective function we have

$$\begin{aligned} E(\log_2|I+R^{-1}HQH^*|) &= \\ E(\log_2|I+R^{-1}(HF)\Lambda(HF)^*|) & \end{aligned} \quad (3)$$

Table 1. Closed formula versus Monte Carlo: $M_s=4$ and $N_s=5$

$\Lambda(P_i)$	$\Psi_i(\alpha_i, \Lambda)$ (Monte Carlo)	$\Psi_i(\alpha_i, \Lambda)$ (Closed formula)
1.0	0.7210	0.7219
3.0	0.3619	0.3613
7.0	0.1804	0.1804
10.0	0.1312	0.1312

Algorithm:

Initialization: $U > 0, V_k > 0, P_i > 0$

Where U and V_k are Lagrangian Multipliers.

Repeat

1. Update F
2. Repeat

$$P_i = (P_i + t_1 * (\Psi_i(\alpha_i, \Lambda) - U - \sum_i V_k \|G_k f_i\|))$$

Until all P_i converge

3. Update

$$U = (U + t_2 * (P_T - \sum_{i=1}^{M_s} P_i))$$

$$V_k = (V_k + t_2 * (\|G_k f_i\|^2 P_i))$$

Until U, V_k converge

With the value of $(\Psi_i(\alpha_i, \Lambda))$ and F , the gradient of the maximization objective function can be solved, and optimal P_i can also be calculated

B. Water Filling Algorithm

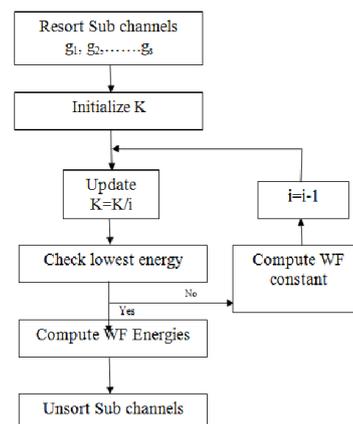


Fig. 2 Flow diagram of Water filling algorithm.

Although Lagrangian Multiplier Algorithm can find a nearly optimal solution it is very complex. In this section we intend to propose a low complexity algorithm to solve another bound maximization problem with (Gastpar.M 2007). Fig. 2 shows the flow diagram of water filling algorithm which is based on channel power or energy.

Unlike the upper bound given by (4), this upper bound is derived from both the statistical expectation of H and R^{-1} . It can make the optimization problem easier.

We get a bound maximization problem as

$$\text{maximize } \sum_i \|G_{k_i}\| P_i < P_T$$

$$P_i > 0 \text{ for all } i$$

Water filling Algorithm Steps:

We do not need to reorder the MIMO-OFDM sub channel gain realization in a descending order.

- Take the inverse of the channel gains.
- Water filling has non uniform step structure due to the inverse of the channel gain.
- Initially take the sum of the Total Power P_t and the Inverse of the channel gain. It gives the complete area in the water filling and inverse power gain.
- Decide the initial water level by the formula given below by taking the average power allocated (average water Level)
- The power values of each sub channel are calculated by subtracting the inverse channel gain of each channel.
- In case the Power allocated value becomes negative stop the iteration process.

Algorithm:

Initialization

Number of Channels;

$$m=N_t=N_r=2 \text{ and } N_1=1, N_2=7;$$

Total transmitted power;

$$P_t=10 \text{ dB}, 20\text{dB}, 30\text{dB}.$$

Equal power distribution

$$P_i = P_t / m; \quad i = 1, 2$$

$$\text{Capacity } C=B * \log_2(1+P_i) \text{ bits/sec.}$$

Waterfilling capacity

$$C_{wf} = \frac{1}{2} * \sum_{i=1}^m \log_2(1 + P_i / N_i) \text{ bits/sec.}$$

Since the Lagrangian dual problem holds the strong duality, Water filling Algorithm can converge to the optimal point within a small range.

IV. SIMULATION RESULTS

In this section, we consider the cognitive radio network where a secondary transmitter-receiver pair coexists with two primary transmitter-receiver pairs. The antenna configuration of the secondary system is set as $M_s = N_s = 2$. We change P_t/N_0 at the secondary transmitter from 0 to 30 dB. Numerical results are given to compare the average capacity of Shannon's capacity, MIMO with $N_T=N_R=2$, Lagrangian multiplier algorithm and Waterfilling algorithm. Here four different STBC codes are taken for wireless channel for simulation.

Symbol Error Rate (SER) for fading channels are calculated based on Space Time Block Codes and Modulation schemes such as QAM, BPSK, QPSK and MFSK and the simulated results are compared with the theoretical values. The average symbol energy E_{avg} for QAM modulation is given by,

$$E_{avg} = \frac{1}{M} \sum_{i=1}^M (a_i^2 + b_i^2) \quad (6)$$

With the definition of energy in mind, symbol error is approximated by,

$$P_e \cong 2 \left(1 - \frac{1}{\sqrt{M}} \right) \text{erfc} \left(\sqrt{\frac{2E_{avg}}{2(M-1)N_o}} \right) \quad (7)$$

Where E_{avg} is calculated by equation (8).

The Symbol error rate for QPSK is given by,

$$P_e = \text{erfc} \left(\sqrt{\frac{E}{2N_o}} \right) \quad (8)$$

This brings up the distinction between symbol error and bit error.

The Symbol error rate for BPSK modulation is given by,

$$P_{e1} = P \left\{ \sqrt{E_b} + n < 0 \mid 1 \right\} \quad (9)$$

Where n is Gaussian with mean 0 and variance $N_o/2$.

The error expression for MFSK modulation with the usual notation is given by,

$$P_e \leq \frac{1}{2} (M - 1) \text{erfc} \left(\sqrt{\frac{E}{2N_o}} \right) \quad (10)$$

MFSK is different from MPSK in that each signal sits on an orthogonal axis (basis).

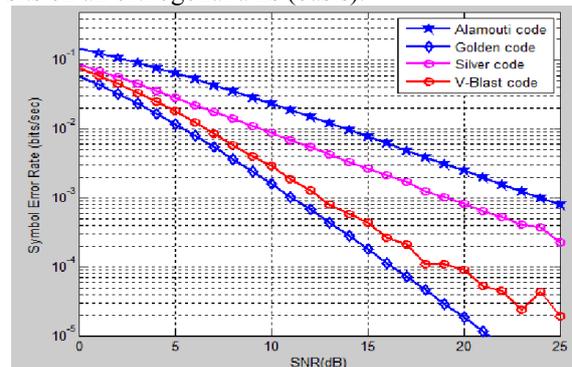


Fig. 3 Symbol Error Rate for Rayleigh Fading Channel using QAM for different STBC Techniques

Fig. 3 shows the Symbol Error Rate (SER) for Rayleigh fading channel using QAM for different STBC techniques. From the analysis of the result, Golden code has minimum SER at SNR=20dB.

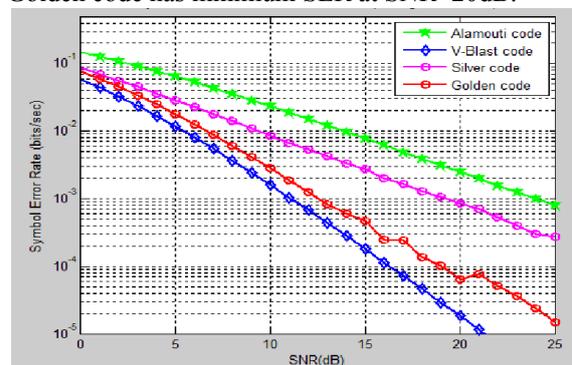


Fig. 4 Symbol Error Rate for Nakagami Fading Channel using QAM for different STBC Techniques

Fig. 4 shows the Symbol Error Rate (SER) for Nakagami fading channel using QAM for different STBC techniques. From the analysis of the result, V-Blast code has minimum SER at SNR=20dB.

Fig. 5 shows the Symbol Error Rate (SER) for Rician fading channel using QAM for different STBC techniques. From the analysis of the result, Silver code has minimum SER at SNR=20dB.

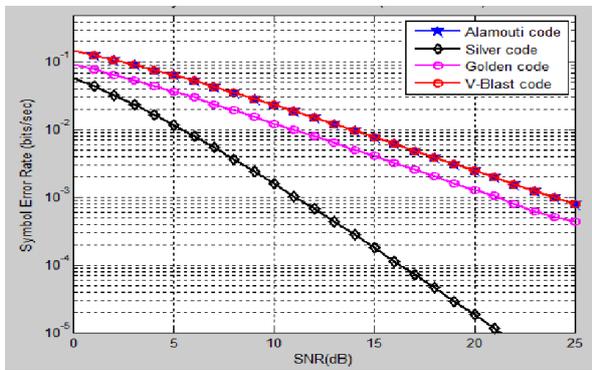


Fig. 5 Symbol Error Rate for Rician Fading Channel using QAM for different STBC Techniques

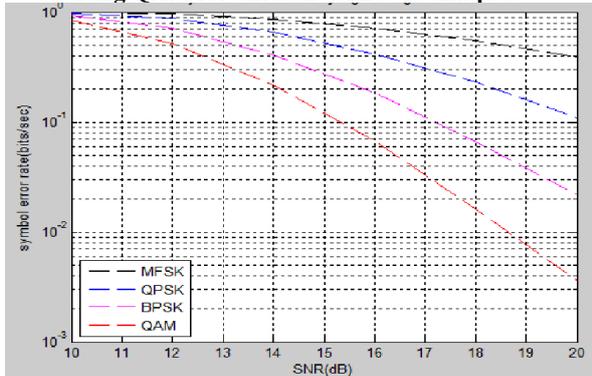


Fig. 6 Performance of Symbol Error Rate for Golden code under different modulations for Rayleigh Fading Channel

Fig. 6 shows shows the performance of Symbol Error Rate (SER) for Golden code under different modulations for Rayleigh fading channel. From the analysis of the result, QAM modulation has minimum SER upto SNR=30dB.

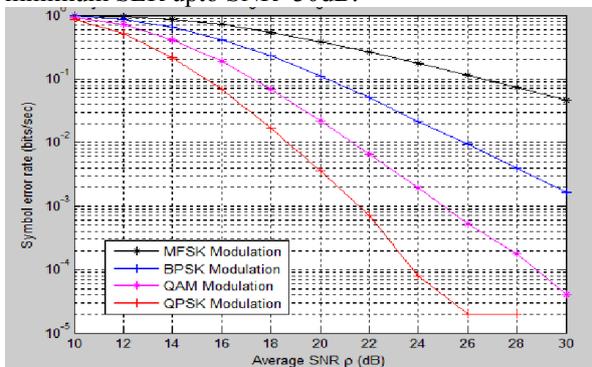


Fig. 7 Performance of Symbol Error Rate for V-Blast under different modulations for Nakagami Fading channel

Fig. 7 shows the performance of Symbol Error Rate (SER) for V-Blast under different modulations for Nakagami fading channel. From the analysis of the result, QPSK modulation has minimum SER upto SNR=30dB.

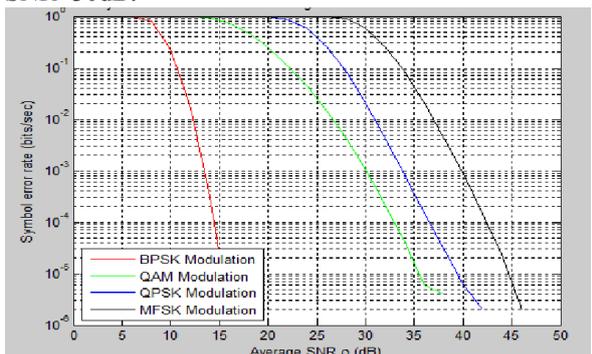


Fig. 8 Performance of Symbol Error Rate for Silver code under different modulations for Rician Fading channel

Fig. 8 shows the performance of Symbol Error Rate (SER) for Silver code under different modulations for Nakagami fading channel. From the analysis of the result, BPSK modulation has minimum SER upto SNR=16dB.

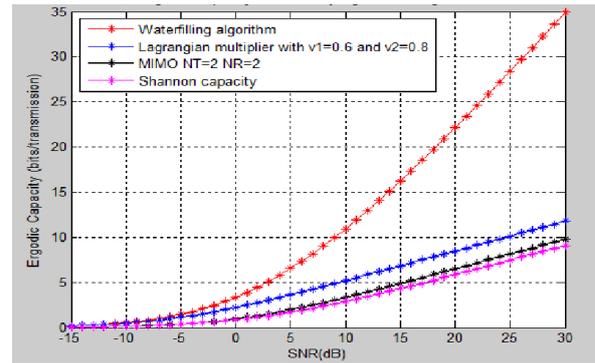


Fig. 9 Comparison of the average capacity of different algorithms for Rayleigh channel with $M_p = N_p = 2$.

Fig 9 shows comparisons between Shannon's capacity, Lagrangian multiplier and water filling algorithm for Rayleigh fading channel. Lagrangian multipliers are initialized with $U=0.6$ and $V_k=0.8$. Water filling algorithm is performed for MIMO with two transmitters and two receivers. By comparing these algorithms water filling algorithm reaches maximum capacity.

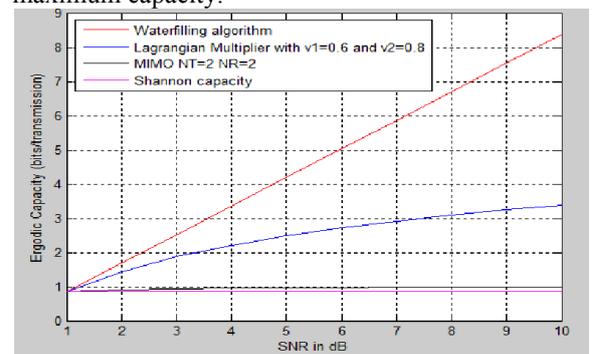


Fig. 10 Comparison of the average capacity of different algorithms for Nakagami channel with $M_p = N_p = 2$.

Fig 10 shows comparisons between Shannon's capacity, Lagrangian multiplier and water filling algorithm for Nakagami fading channel. Lagrangian multipliers are initialized with $U=0.6$ and $V_k=0.8$. Water filling algorithm is performed for MIMO with two transmitters and two receivers. By comparing these algorithms water filling algorithm reaches maximum capacity

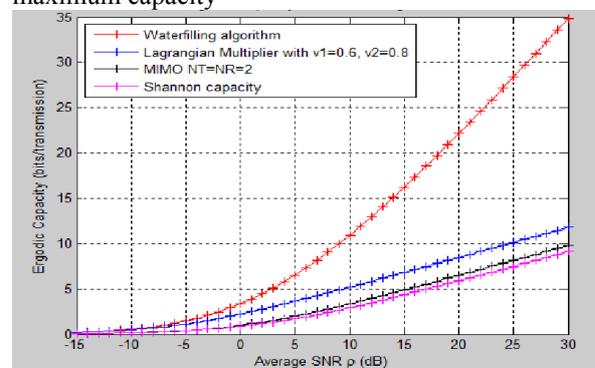


Fig. 11 Comparison of the average capacity of different algorithms for Rician channel with $M_p = N_p = 2$.

Fig 11 shows comparisons between Shannon's capacity, Lagrangian multiplier and water filling algorithm for Rician fading channel respectively. Lagrangian multipliers are initialized with $U=0.6$ and $V_k=0.8$. Water filling algorithm is performed for MIMO with two transmitters and two receivers. By

comparing these algorithms water filling algorithm reaches maximum capacity. Table 2 compares the channel capacity of the system without using STBC and with using STBC codes. From that we clearly understand STBC with different modulation schemes gives very good performance in channel capacity for all the three wireless channels.

Table 2 Compares the Channel Capacity of previous and proposed algorithms.

Channel Type	Channel Capacity without STBC and Modulation Schemes (bits/s/Hz)	Channel Capacity with STBC and Modulation Schemes (bits/s/Hz)
Rayleigh Fading Channel	25	35
Nakagami Fading Channel	5	9
Rician Fading Channel	20	35

V. CONCLUSION

Space time block code (STBC) is a technique used in wireless communication to transmit multiple copies of data stream across a number of antennas. In this paper, with the assumption of primary communication and secondary communication are going in a simultaneous manner. Based on the STBC code, the symbol error rate (SER) is calculated for various modulation schemes. According to the SER, Lagrangian multiplier algorithm and water filling algorithm are proposed to maximize the ergodic capacity.

From the performance analysis, we find that water filling algorithm can approach maximum capacity. Further it is recommended for practical systems because it can achieve the better tradeoff between complexity of algorithm and throughput performance.

REFERENCES

1. Caire.G, Taricco.G, and Biglieri.E.,1999, "Optimum power control over fading channels," IEEE Trans. Inform. Theory, Vol. 45, no. 5, pp. 1468–1489.
2. Chengshan Xiao, Yahong Rosa Zheng and Norman C. Beaulieu, 2006, " Novel Sum of Sinusoids Simulation Models for Rayleigh and Rician fading Model", IEEE Transaction on Wireless Communication", Vol..5, No.12.
3. Dongming Wang, Jiangzhou Wang, Xiaohu You, Yan Wang,Ming Chen, and Xiaoyun Hou, 2013, " Spectral Efficiency of Distributed MIMO Systems", Journal on Selected areas in Communications, Vol. 31, No. 10.
4. Gastpar.M, 2007, "On capacity under receive and spatial spectrum-sharing constraints," IEEE Trans. Inform. Theory, Vol. 53,No. 2, pp. 471–487.
5. Ilesanmi Banjo Oluwafemi, 2013, "Improved superorthogonal space time trellis coded MIMO OFDM system", IETE Journal of Research (Impact Factor: 0.2).Vol.59, No.6, pp.665-671.
6. Junling Mao, Jinchun Gao, Yuanan Liu, Gang Xie, and Xiuwen Li, 2012, " Power Allocation Over Fading Cognitive MIMO Channels: An Ergodic Capacity Perspective", IEEE Transactions on Vehicular Technology, Vol. 61, No. 3.
7. Liang.Y.C., Zhang.R., and Cioffi.J, 2006, "Subchannel grouping and statistical waterfilling for vector block-fading channels," IEEE Trans. Commun., Vol. 54, No. 6, pp. 1131–1142.
8. Mitola.J and Maguire.G.Q., 1999, "Cognitive radio: Making software radios more personal," IEEE Pers. Commun., Vol. 6,No. 6, pp. 13–18.
9. Musavian.L, and Aissa,S, 2007, "Ergodic and outage capacities of spectrum-sharing systems in fading channels," in Proc. IEEE Global Telecommunications

Conference (GLOBECOM07), Washington. DC, USA, pp. 3327–3331.

10. Yingbin Liang and Venugopal V. Veeravalli, 2004," Capacity of Non coherent Time Selective Rayleigh Fading Channel", IEEE Transaction on Information Theory, Vol.50, No.12.
11. Zhang,L, Liang,,Y.C. and Xin, .Y, 2008, "Joint beam forming and power allocation for multiple access channels in cognitive radio networks," IEEE J. Select. Areas Communication., Vol. 26, No. 1, pp. 38–51,...
12. Zhang Q.T., 2003, " A Generic Correlated Nakagami Fading Model for Wireless Communication", IEEE Transaction on Communication, Vol.51, No.11.