

ESTIMATION OF INDUCTANCE GRADIENT EMPIRICAL FORMULA OF RAILS USING REGRESSION ANALYSIS TECHNIQUE

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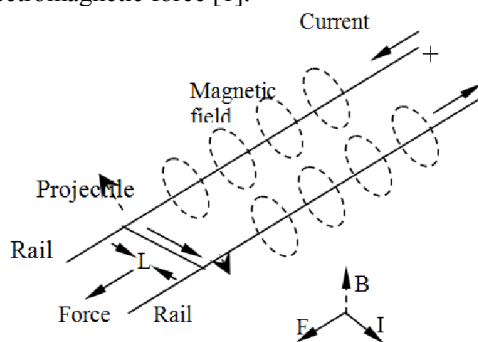
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¹Professor, Department of Electrical and Electronics Engineering, Easwari Engineering College, Chennai, India²Professor - EEE, Faculty of Engg, Karpagam academy of Higher Education, Coimbatore³ Professor, Electrical Engineering, Imam Khomeini international University, (IKIU) Iran⁴ Imam Khomeini International University, Qazvin, Iran**ABSTRACT**

Rail gun is an electromagnetic apparatus that translates the electrical energy into kinetic energy to accelerate the projectile along the rails with higher velocity. The fraction of kinetic energy to electrical energy is called efficiency of the rail gun. The efficiency of the rail gun depends on inductance gradient of the rails (L'), as it straight away decides the force to current that accelerates the projectile. For the past several years the L' values are calculated using analytical methods and numerical methods. The analytical method is suitable for solving simple problems and numerical method needs code and program and also it is a time consuming process. Hence, a simple method is needed to calculate L' values of the rails. As the L' values depends on rail dimensions, now-a-days, researchers focuses on obtaining a simple empirical formula which can be used to compute the L' values of the rails with respect to rail dimensions. In this paper, an attempt is made to extract an empirical formula that can be used to compute L' of the rails using regression analysis technique. In order to extract an empirical formula, the L' values are calculated using ANSOFT field simulator for various rail dimensions initially. The obtained L' values are entered and simulated by using curve fitting software called Oakdale Data Fit Engineering and the best equation, which can be used to compute the L' of the rails extracted. The L' values are calculated using the extracted empirical formula and compared with the other researcher's values shown a good agreement between the results.

Keywords: Inductance gradient, regression analysis, Formula**I. INTRODUCTION**

The rail gun is a device which converts the electrical energy into mechanical energy for accelerating the projectile to hypervelocity. It consists of two parallel conductors called the rails that are bridged by conductor called projectile. A simple graphic representation rail gun is shown in Fig1. The current flows through one rail, passes through the projectile perpendicular to the rails, and passes through other rail in parallel. As a result, the two rails produce the magnetic field and an intercepting field is produced by the armature. The rails repel each other and they both in turn repel the armature. Since the rails are both fixed, the net result is a propulsive force on the armature, which will be accelerated by electromagnetic force [1].

**Fig.1 A simple graphic representation of rail gun [1]**

The force acting on the projectile is depends on inductance gradient of the rails and it determines the efficiency of the rail gun. Acceleration force and L' could be examined instead of efficiency [2]. These are related by the following equation

$$\vec{F} = \frac{1}{2} L I^2 \quad (1)$$

Where:

 L' is the inductance gradient of the rails in $\mu\text{H}/\text{m}$ I is the current through armature in Amps F is the force acting on the armature in Newton

Jerry F. Kerrisk has calculated the inductance gradient values of the rails with respect to the rail dimensions using A.C with high frequency limits using finite difference method [3]. Honjo has

calculated the force acting on the rail conductors using BUS 3D program [4]. Huerta has calculated the inductance gradient of the rails using conformal mapping method [5]. From the literature review it is observed that for the past several years various numerical method and analytical method were developed to compute the inductance gradient of the rails but the analytical method is suitable for solving simple problems and numerical method needs code and program and also it is a time consuming process. Hence, a simple method is needed to calculate L' values of the rails. As the L' values depends on rail dimensions, now-a-days, researchers focuses on obtaining a simple empirical formula which can be used to compute the L' values of the rails with respect to rail dimensions[6-7]. In this paper, an effort is made to extract an empirical formula that can be used to compute L' of the rails using regression analysis technique.

II. INTRODUCTION TO REGRESSION ANALYSIS

Regression is a numerical measures those efforts to decide the strength of the connection between one dependent variable and a single or a series of other altering variables. In statistics, regression analysis is an arithmetical method for approximation the relationships among variables. It takes many techniques for modeling and analyzing several variables, when the focus is on the relationship between a dependent variable and one or more independent variables. The process of getting a regression equation consists in postulating a type of the function to be fitted, $Y = f(X)$, which will depend, in general, on a number of parameters, say $b_0; b_1; \dots; b_k$. Then we select a appropriate method to determine the values of those parameters. The most commonly used method for this purpose is the Least Squares Method[11]. The least square method assumes that the best fit curve for the given data has the minimal sum of the deviations squared error from the given set of data.[9] In regression analysis technique the dependent variables is affected by more than one independent variables or there may

be more than one independent variable will be changed at the same time.

An example for the two independent variables is x , y and one dependent variable z and the linear relationship is given as

$$z = a + bx + cy \tag{2}$$

For a given data set $x_1, y_1, z_1, x_2, y_2, z_2, x_3, y_3, z_3, \dots, x_n, y_n, z_n$ where $n \geq 2$, the best fitting curve $f(x)$ has the least square error, i.e.,

$$\Pi = \sum_{i=1}^n (z_i - f(x_i))^2 = \sum_{i=1}^n (z_i - (a + bx_i + cy_i))^2 = \min \tag{3}$$

Here a, b and c are unknown constant when all x_i, y_i and z_i is given. To obtain the least square error, the unknown coefficient a, b and c must yield zero first derivatives.

$$\frac{\partial \Pi^2}{\partial a} = \sum_{i=1}^n (z_i - (a + bx_i + cy_i)) = 0 \tag{4}$$

$$\frac{\partial \Pi^2}{\partial b} = 2 \sum_{i=1}^n x_i (z_i - (a + bx_i + cy_i)) = 0 \tag{5}$$

$$\frac{\partial \Pi^2}{\partial c} = 2 \sum_{i=1}^n y_i (z_i - (a + bx_i + cy_i)) = 0 \tag{6}$$

Expanding the above equations, we have

$$\sum_{i=1}^n z_i = a \sum_{i=1}^n 1 + b \sum_{i=1}^n x_i + c \sum_{i=1}^n y_i \tag{7}$$

$$\sum_{i=1}^n x_i z_i = a \sum_{i=1}^n x_i + b \sum_{i=1}^n x_i^2 + c \sum_{i=1}^n x_i y_i \tag{8}$$

$$\sum_{i=1}^n y_i z_i = a \sum_{i=1}^n y_i + b \sum_{i=1}^n x_i y_i + c \sum_{i=1}^n y_i^2 \tag{9}$$

The unknown coefficients a, b and c will be obtained by solving the above linear equations. After finding unknown constants, π is calculated using the equation (3). If the π value is not minimum, unknown variable is added into the equation (2) and π is calculated until the minimum least square error is obtained.

III. EMPIRICAL EQUATION OF INDUCTANCE GRADIENT OF RAIL FOR RECTANGULAR GEOMETRY

Fig.2 shows two dimensional view of rectangular bore geometry. The L' values are depends on rail

$$L' = a + b \left(\ln \left(\frac{S}{H} \right) \right) + c \left(\ln \left(\frac{W}{H} \right) \right) + d \left(\ln \left(\frac{S}{H} \right) \right)^2 + e \left(\ln \left(\frac{W}{H} \right) \right)^2 + f \left(\ln \left(\frac{S}{H} \right) \ln \left(\frac{W}{H} \right) \right) + g \left(\ln \left(\frac{S}{H} \right) \right)^3 + h \left(\ln \left(\frac{W}{H} \right) \right)^3 + i \left(\ln \left(\frac{S}{H} \right) \ln \left(\frac{W}{H} \right) \right)^2 + j \left(\ln \left(\left(\frac{S}{H} \right) \right)^2 \ln \left(\frac{W}{H} \right) \right) \quad \mu\text{H/m} \tag{10}$$

Where $a, b, c, d, e, f, g, h, i,$ and j are regression variables. The regression variable values are given in Table II. In Table II, the lower limits and upper limits of each variable values are given. In order to get accurate value of L' , exact value of the regression variables are chosen between the lower limits and upper limits. The exact value of regression variables are also given in the same table.

The L' values obtained from the regression analysis equation, for different rail ratios, are compared with other researcher's values and given in Table III.

From the table, it is observed that L' values obtained using regression analysis equation shows a good

agreement with other researcher's values. From the table, it is also observed that L' values obtained using IEM method (7) equation shows a good agreement with other researcher's values, for a lower value of S/H and higher value of W/H and S/H of rail dimensions. For a higher value of S/H and lower value of W/H and for a higher value of W/H and lower value of S/H , the L' values obtained using IEM method equation shows a more deviation in values as compared with regression analysis technique.

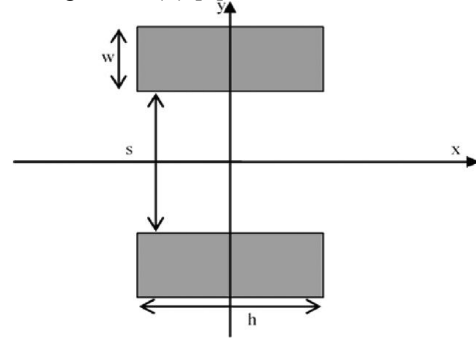


Fig.2 Two dimensional view of rail gun [7]

The rails are assumed to be made up of copper with conductivity equal to $5.8 \times 10^7 \text{ S/m}$ and carrying a time varying current in the high frequency limit. In order to calculate the L' of rails, rail dimensions is varied. Murugan et al [8] has already obtained the inductance gradient values of rectangular geometry using finite element method in A.C high frequency limit using ANSOFT field simulator. It calculates the magnetic field distribution in a space using Maxwell's equations [9]. The inductance gradient values, which obtained using ANSOFT field simulator by Murugan et al, are not sufficient to extract the empirical formula to compute the L' values of the rails. Hence once again the inductance gradient values for various rail dimensions are calculated using ANSOFT field simulator. Different ratios of rail Width to rail Height (W/H) and rail Separation to rail Height (S/H) are taken for calculating the L' values of the rails. The S/H and W/H ratios are varied from 0.1 to 1 in step of 0.1 and the inductance gradient values of the rails are calculated and given in Table I.

The inductance gradient values obtained from the ANSOFT field simulation are entered into curve fitting software called Oakdale date fit engineering and simulated. It gives 254 possible empirical formulas with regression variables for the given set of data's. For each and every formula, L' values are calculated by substituting regression variables in to the equation. Finally it's found that equation (10) gives less deviation in L' values and chosen as best empirical formula which could be used to compute the inductance gradient value of rails.

TABLE I. INDUCTANCE GRADIENT VALUE FOR VARIOUS RAIL DIMENSIONS($\mu\text{H}/\text{m}$)

S/H	W/H									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1	0.558	0.536	0.519	0.505	0.493	0.482	0.473	0.465	0.458	0.4517
0.9	0.526	0.505	0.489	0.476	0.465	0.455	0.446	0.439	0.432	0.425
0.8	0.491	0.472	0.457	0.445	0.434	0.428	0.417	0.410	0.404	0.398
0.7	0.453	0.436	0.422	0.411	0.402	0.393	0.386	0.379	0.373	0.368
0.6	0.413	0.396	0.384	0.374	0.364	0.357	0.352	0.345	0.340	0.335
0.5	0.366	0.352	0.341	0.332	0.325	0.318	0.312	0.307	0.302	0.298
0.4	0.315	0.302	0.293	0.286	0.279	0.273	0.269	0.264	0.260	0.257
0.3	0.256	0.246	0.238	0.232	0.227	0.223	0.219	0.215	0.212	0.209
0.2	0.187	0.180	0.175	0.170	0.166	0.163	0.160	0.158	0.156	0.154
0.1	0.105	0.101	0.098	0.096	0.094	0.092	0.090	0.089	0.088	0.087

TABLE II REGRESSION VARIABLES VALUES

Variable	Lower Limit	Upper Limit	Selected value of constants
a	0.45025	0.45322	0.45173
b	0.24549	0.25463	0.25006
c	-6.949×10^{-02}	-6.035×10^{-02}	-6.492×10^{-02}
d	3.753×10^{-02}	4.6792×10^{-02}	4.2161×10^{-02}
e	-9.174×10^{-03}	8.798×10^{-03}	-4.543×10^{-03}
f	-0.04243	-0.03554	-3.8983×10^{-02}
g	-3.488×10^{-04}	2.298×10^{-03}	9.748×10^{-04}
h	0.00051	3.155×10^{-03}	1.831×10^{-03}
i	-5.244×10^{-03}	-3.0764×10^{-03}	-4.1602×10^{-03}
j	-6.897×10^{-03}	-4.7299×10^{-03}	-5.8136×10^{-03}

TABLE III COMPARISON OF INDUCTANCE GRADIENT BETWEEN THE RESEARCHERS

Rail dimension s	L' ($\mu\text{H}/\text{m}$) from FEM	L' ($\mu\text{H}/\text{m}$) From other researchers	Comparison of L' using other empirical Formulae		
			L' ($\mu\text{H}/\text{m}$)[6]	L' ($\mu\text{H}/\text{m}$)[7]	L' value from regression analysis Equation ($\mu\text{H}/\text{m}$)
S/H = 1 W/H = 1	0.4517	0.45174[2]	0.4517	0.4517	0.4517
S/H = 0.6 W/H = 0.5	0.3643	0.364[5]	0.3675	0.3098	0.3658
S/H = 10 W/H = 10	-	0.7	0.746	0.742	0.745
S/H = 10 W/H = 0.1	-	-	1.4106	1.3451	1.5928
S/H = 0.5 W/H = 10	-	-	0.2386	0.26983	0.2256

It is also important to note that the L' values used in regression analysis technique, to extract L' empirical formula, are calculated only for a lower values of S/H and W/H of rail dimensions. After extracting the L' empirical formula using regression analysis technique, L' values are also computed for a higher value of S/H and W/H of rail dimensions and compared with other researchers values shows a good agreement between the results.

To analyze the extracted empirical formula in more detail, L' values are calculated using the empirical formula for various rail dimensions. Four cases have been taken to calculate L' of the rails.

Case1. Higher values of S/H and W/H of rail dimension.

(The value of S/H is varied from 1 to 10 in steps of 1 for the ratio of W/H = 10)

Case2. Lower values of S/H and W/H of rail dimension.

(The value of S/H is varied from 0.1 to 1 in steps of 0.1 for the ratio W/H = 0.1)

Case3. Higher values of S/H and lower values of W/H of rail dimension.

(The value of S/H is varied from 1 to 10 in steps of 1 for the ratio of W/H = 0.1)

Case4. Lower values of S/H and higher values of W/H of rail dimension.

(The value of S/H is varied from 0.2 to 1 in steps of 0.2 for the ratio of W/H = 10)

The L' values are computed for the above cases using the extracted empirical formula and graphs are plotted as shown in the Fig. 3 to Fig.6.

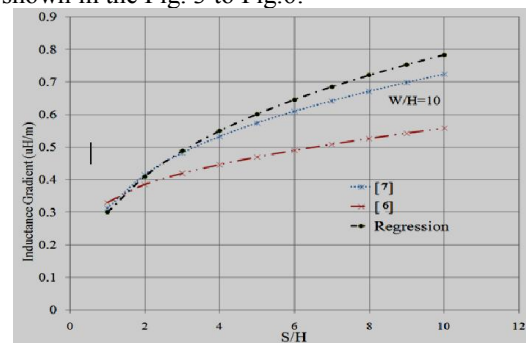


Fig. 3 Inductance gradient values for Case 1

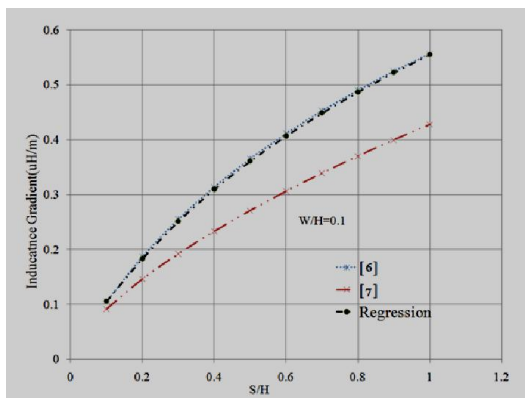


Fig. 4 Inductance gradient values for Case 2

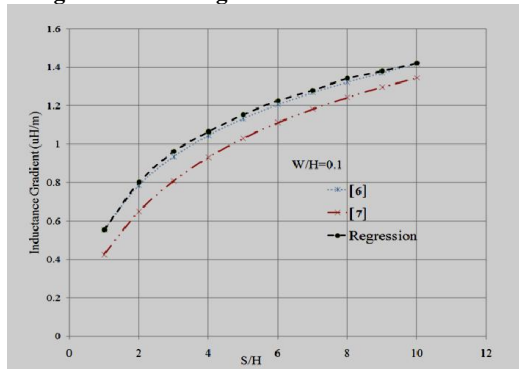


Fig. 5 Inductance gradient values for Case 3

From the figures, it is observed that the L' values obtained using regression analysis and IEM method empirical formulae, for higher values of S/H and W/H, and lower values of S/H and W/H of rail

$$L' = a + b \left(\ln \left(\frac{S}{H} \right) \right) + c \left(\ln \left(\frac{W}{H} \right) \right) + d \left(\ln \left(\frac{S}{H} \right) \right)^2 + e \left(\ln \left(\frac{W}{H} \right) \right)^2 + f \left(\ln \left(\frac{S}{H} \right) \ln \left(\frac{W}{H} \right) \right) + g \left(\ln \left(\frac{S}{H} \right) \right)^3 + h \left(\ln \left(\frac{W}{H} \right) \right)^3 + i \left(\ln \left(\frac{S}{H} \right) \right) \left(\ln \left(\frac{W}{H} \right) \right)^2 + j \left(\ln \left(\frac{S}{H} \right) \right)^2 \ln \left(\frac{W}{H} \right) \quad \mu\text{H/m} \quad (11)$$

IV. CONCLUSION

In this paper, work have been carried out to extract an empirical formula that can be used to compute L' of the rails using regression analysis technique. In rectangular bore geometry the L' values are calculated using FEM simulation for various values of rail dimensions. These values are entered into a curve fitting software and L' empirical formula has been extracted. The L' values are calculated using the extracted formula for various values of rail dimensions and the following conclusions have been made using the obtained data's.

1. The L' values obtained using regression analysis empirical formula has been compared with other researcher's values and have shown a good agreement between the results.
2. It has been observed that the L' values obtained using IEM empirical formula have shown a good agreement with other researches values for lower values of S/H and W/H, and higher values of S/H and W/H of rail dimensions. But L' values obtained for higher values of S/H and lower values of W/H, and lower values of S/H and higher values of W/H of rail dimensions have shown a more error in value as compared with regression analysis technique. Then, we offer formula (11) as a new empirical formula for calculating L' of rectangular rail gun.

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dimensions (Fig. 3 and Fig. 4) shows a good agreement with other researchers values.

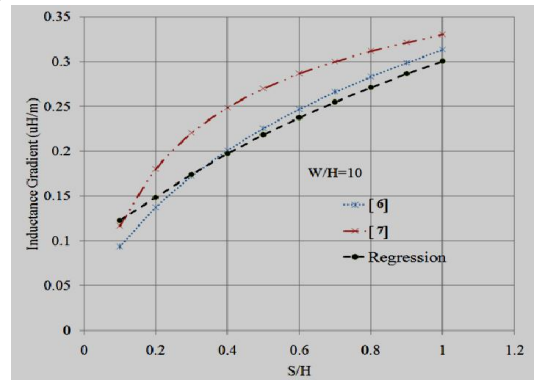


Fig. 6 Inductance gradient values for Case 4

In the case of higher values of S/H and lower values of W/H and lower values of S/H and higher values of W/H of rail dimensions (Fig.5 and Fig.6) , the L' values obtained using IEM method equation shows more deviation in values as compared with regression analysis equation. So, it is concluded that the L' empirical formula which is obtained using IEM method can be applicable for lower values of S/H and W/H, and higher values of S/H and W/H of rail dimensions. But, the empirical formula which is obtained using regression analysis technique can be applicable for any values of W/H and S/H of rail dimensions. So we offer the following empirical formula to calculate L' of the rectangular rail gun.

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